Assignment #7

Due on Wednesday, October 17, 2012

Read Section 5.1, *The Area Problem,* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Background and Definitions

- *The Area Function for Non–Negative Functions.* Let f denote a non–negative, real–value, piece–wise continuous function, and a a point in the domain of f contained in an interval. Suppose that $x > a$ and consider the region, R, in the ty–plane lying above the t–axis, below the graph of $y = f(t)$, and between the vertical lines $t = a$ and $t = x$. We denote the area of that region by $A_f(a; x)$. If $x < a$, we set $A_f(a; x)$ to be the negative of the area of the region R. We call $A_f(a; x)$ the area function of f from a to x.

- *The Area Function for Sign–Changing Functions.* Let f denote a piecewise continuous real–value function, and a a point in the domain of f contained in an interval. The area function of f from a to x is the number obtained by computing the area of the region, R, in the ty–plane bounded by t–axis, the graph of $y = f(t)$, and between the vertical lines $t = a$ and $t = x$ and following the following sign convention:

  (i) area of regions below the t–axis are taken to be negative;

  (ii) x is to the right of a, the result of the area calculation is multiplied by $+1$; when x is to the left of a the result is multiplied by -1.

Do the following problems

1. Let $f(t) = |t - 2|$ for all $t \in \mathbb{R}$ and $a = -1$.

   (a) Sketch the graph of $y = f(t)$ in the ty–plane.

   (b) Using your knowledge of areas of triangles, compute $A_f(a; x)$ and sketch the graph of A_f as function of x in the xy–plane.

2. Let $f(t) = \begin{cases} 0, & \text{for } t < 2; \\ 1, & \text{for } t \geqslant 2, \end{cases}$ and $a = -1$.

   (a) Sketch the graph of $y = f(t)$ in the ty–plane.

   (b) Using your knowledge of areas of rectangles, compute $A_f(a; x)$ and sketch the graph of A_f as function of x in the xy–plane.

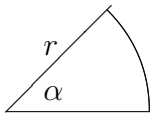

Figure 1: Circular Sector

3. *Area of a Circular Sector.* Figure 1 shows the sector of a circle of radius r subtended by an angle of α from its center. Use the procedure outlined in the class lecture notes to come up with a formula for computing the are of the sector in terms of r and the angle α.

4. Let f denote the function defined by $f(t) = \sqrt{1-t^2}$ for $-1 \leqslant t \leqslant 1$.

   (a) Sketch the graph of $y = f(t)$ in the ty–plane.

   (b) Use the result of Problem 3 and your knowledge of areas of triangles in order to compute $A_f(0; x)$ for $-1 \leqslant x \leqslant 1$.

5. Let f be a real valued function defined on the real line by the formula

$$f(t) = \begin{cases} 0, & \text{if } t < -1; \\ t+1, & \text{if } -1 \leqslant t < 1; \\ 2, & \text{if } t \geqslant 1. \end{cases}$$

   Let $A_f(-1; x)$ denote the area function for f from -1 to x. Give a formula for computing $A_f(-1; x)$ for all values of x.