Review Problems for Exam 1

1. Suppose that water leaks out a barrel at a constant rate. If the water level starts at 36 inches and drops to 35 inches in 30 seconds, how long will it take for the water to leak out of the barrel?

2. Derive the identity $\dfrac{1}{n(n+1)} = \dfrac{1}{n} - \dfrac{1}{n+1}$, and use it to compute

$$\lim_{n\to\infty} \frac{1}{n(n+1)}.$$

3. Use the fact that $\displaystyle\lim_{\theta\to 0} \frac{\sin(\theta)}{\theta} = 1$ to evaluate the following limits:

   (a) $\displaystyle\lim_{t\to 0} \frac{\tan(t)}{t}$

   (b) $\displaystyle\lim_{t\to 0} \frac{\sin^2(t)}{t}$

4. Evaluate the following limits:

   (a) $\displaystyle\lim_{t\to 0} \frac{t^2 - 1}{t+1}$

   (b) $\displaystyle\lim_{t\to\pi} \frac{\cos^2(t)}{t}$

   (c) $\displaystyle\lim_{t\to -2} \frac{t+1}{t-1}$.

5. Explain why each of the given functions, f, are continuous on $\mathbb{R}$.

   (a) $f(t) = \dfrac{\sin^3 t}{1+t^2}$ for all $t \in \mathbb{R}$.

   (b) $f(t) = \begin{cases} t^2 \sin\left(\dfrac{1}{t}\right) & \text{if } t \neq 0; \\ 0 & \text{if } t = 0. \end{cases}$

6. For the given function f, discuss the continuity or discontinuity of f at the given point a.

   (a) $f(t) = |t-2|$ for all $t \in \mathbb{R}$ and $a = 2$.

   (b) $f(t) = \begin{cases} 1 & \text{if } t < 0; \\ -1 & \text{if } t \geqslant 0, \end{cases}$ and $a = 0$.