Topics for Exam 3

1. **Interpretations of the Riemann Integral**
   - 1.1 The area function
   - 1.2 Recovering a function from its rate of change
   - 1.3 Computing the amount of a substance from its linear density
   - 1.4 Average of a function
2. **Differential Calculus**
   - 2.1 Definition of differentiable function
   - 2.2 The derivative
   - 2.3 Properties of differentiable functions
     - 2.3.1 Multiples and sums of differentiable functions
     - 2.3.2 Products of differentiable functions
     - 2.3.3 Compositions of differentiable functions
     - 2.3.4 Quotients of differentiable functions
   - 2.4 Interpretations of the derivative
     - 2.1 Instantaneous rate of change
     - 2.2 Linear approximation to a differentiable function
     - 2.3 Tangent line to the graph of a differentiable function.
3. **Fundamental Theorems**
   - 3.1 Recovering a function from its rate of change
   - 3.2 Differentiability of the area function
   - 3.3 Evaluating integrals

Relevant sections in the online lecture notes: 5.4, 6.1, 6.2, 6.3, 6.4, 7.1, 7.2 and 7.3.

Relevant Assignments: 12, 13, 14, 15, 16 and 17.

Important Concepts: Average of a function, linear density, instantaneous rate of change, differentiable function, the derivative, linear approximation to a differentiable function, tangent line to the graph of a differentiable function.

Important Results

- *First Fundamental Theorem of Calculus.* (Theorem 7.1.1 in the Class Notes).

  Let f be a differentiable function defined in an open interval I containing a. Suppose that f' is continuous on I. Then,

  $$f(t) = f(a) + \int_a^t f'(\tau)\ d\tau, \quad \text{for all } t \in I.$$

- *Second Fundamental Theorem of Calculus.* (Theorem 7.2.1 in the Class Notes).

  Let f be a continuous function defined in an open interval I containing a. Then, the function

  $$G(x) = \int_a^x f(t)\ dt, \quad \text{for all } x \in I,$$

  is differentiable in I and $G'(x) = f(x)$ for all $x \in I$, or

  $$\frac{d}{dx}\left[\int_a^x f(t)\ dt\right] = f(x), \quad \text{for } x \in I.$$

- *Third Fundamental Theorem of Calculus.* (Theorem 7.1.1 in the Class Notes).

  Let f be a continuous function defined in an open interval I. Assume that there exists a function, F, that is differentiable in I and $F'(x) = f(x)$ for all $x \in I$. Then, for any $a, b \in I$,

  $$\int_a^b f(t)\ dt = F(b) - F(a).$$

Important Skills: Know how to compute the average value of a function; know how to compute the amount of a substance from its linear density; know how determine if a function is differentiable or not; know how to apply the properties of differentiation to compute derivatives of differentiable functions; know how to compute the linear approximation to a differentiable function at a given point; know how to compute the tangent line to the graph of a differentiable function at any point; know when and how to apply the Fundamental Theorems of Calculus.