Assignment #15

Due on Wednesday, October 30, 2013

Read Section 5.1 on the *Definition of the Joint Distribution* and Section 5.2 on *Marginal Distributions* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 3.4 on *Bivariate Distributions* in DeGroot and Schervish.

Read Section 3.5 on *Marginal Distributions* in DeGroot and Schervish.

Do the following problems

1. Let $F_{(X,Y)}$ be the joint cdf of two random variables X and Y. For real constants $a < b$, $c < d$, show that

   $$\Pr(a < X \leqslant b, c < Y \leqslant d) = F_{(X,Y)}(b,d) - F_{(X,Y)}(b,c) - F_{(X,Y)}(a,d) + F_{(X,Y)}(a,c).$$

   Use this result to show that $F(x,y) = \begin{cases} 1 & \text{if } x + 2y \geqslant 1, \\ 0 & \text{otherwise,} \end{cases}$ cannot be the joint cdf of two random variables.

2. Let $g(t)$ denote a non–negative, integrable function of a single variable with the property that $\displaystyle\int_0^\infty g(t)\ \mathrm{d}t = 1$. Define

   $$f(x,y) = \begin{cases} \dfrac{2g(\sqrt{x^2+y^2})}{\pi\sqrt{x^2+y^2}} & \text{for } 0 < x < \infty,\ 0 < y < \infty, \\ 0 & \text{otherwise.} \end{cases}$$

   Show that $f(x,y)$ is a joint pdf for two random variables X and Y.

3. Let X and Y have joint pdf $f_{(X,Y)}(x,y) = \begin{cases} e^{-x-y} & \text{for } 0 < x < \infty,\ 0 < y < \infty, \\ 0 & \text{otherwise.} \end{cases}$

   Define $Z = X + Y$. Compute $\Pr(Z \leqslant z)$ for $0 < z < \infty$ and give the pdf of Z.

4. Let X and Y have joint pdf $f_{(X,Y)}(x,y) = \begin{cases} 1 & \text{for } 0 < x < 1,\ 0 < y < 1, \\ 0 & \text{otherwise.} \end{cases}$ Find the cdf and pdf of the product $Z = XY$.

5. Suppose that two persons make an appointment to meet between 5 PM and 6 PM at a certain location and they agree that neither person will wait more than 10 minutes for each person. If they arrive independently at random times between 5 PM and 6 PM, what is the probability that they will meet?