**Assignment #16**

**Due on Friday,November 1, 2013**

**Read** Section 5.3 on the *Independent Random Variables* and Section 6.1 on *The Normal Distribution* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

**Read** Section 3.5 on *Marginal Distributions* in DeGroot and Schervish.

**Read** Section 5.6 on *The Normal Distribution* in DeGroot and Schervish.

**Do** the following problems

1. Suppose $X$ and $Y$ are independent and let $g_1(X)$ and $g_2(Y)$ be functions for which $E(g_1(X)g_2(Y))$ exists. Show that

$$E(g_1(X)g_2(Y)) = E(g_1(X)) \cdot E(g_2(Y))$$

Conclude therefore that if $X$ and $Y$ are independent and $E(|XY|)$ is finite, then

$$E(XY) = E(X) \cdot E(Y).$$

2. Suppose $X$ and $Y$ are independent random variables for which the moment generating functions exist on some common interval of values of $t$. Show that

$$\psi_{X+Y}(t) = \psi_X(t) \cdot \psi_Y(t)$$

for $t$ is the given interval.

3. Suppose that $X \sim \text{Normal}(\mu, \sigma^2)$ and define $Y = \dfrac{X - \mu}{\sigma}$.

Prove that $Y \sim \text{Normal}(0, 1)$

4. Let $X_1$ and $X_2$ denote independent, $\text{Normal}(0, \sigma^2)$ random variables, where $\sigma > 0$. Define the random variables

$$\overline{X} = \frac{X_1 + X_2}{2} \quad \text{and} \quad Y = \frac{(X_1 - X_2)^2}{2\sigma^2}.$$

Determine the distributions of $\overline{X}$ and $Y$.

*Suggestion:* To obtain the distribution for $Y$, first show that

$$\frac{X_1 - X_2}{\sqrt{2}\,\sigma} \sim \text{Normal}(0, 1).$$

5. Let $X_1$, $X_2$, $\overline{X}$ and $Y$ be as in the previous problem. Prove that $\overline{X}$ and $Y$ are independent.

*Suggestion:* Start working on this problem as soon as possible!