Assignment #21

Due on Friday, November 22, 2013

Read Section 7.3 on the *Central Limit Theorem* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 6.2 on *The Law of Large Numbers* in DeGroot and Schervish.

Read Section 6.3 on *The Central Limit Theorem* in DeGroot and Schervish.

1. Forty–seven measurements are recorded to several decimal places. Each of these 47 numbers is rounded off to the nearest integer. The sum of the original 47 numbers is approximated by the sum of those integers. Assume that the errors made in rounding off are independent, identically distributed random variables with a uniform distribution over the interval $(-0.5, 0.5)$. Compute approximately the probability that the sum of the integers is within two units of the true sum.

2. Let X denote a random variable with pdf $f_X(x) = \begin{cases} \dfrac{1}{x^2} & \text{if } 1 < x < \infty, \\ 0 & \text{otherwise.} \end{cases}$ Consider a random sample of size 72 from this distribution. Compute approximately the probability that 50 or more observations of the random sample are less than 3.

3. How large a random sample must be taken from a given distribution in order for the probability to be at least 0.99 that the sample mean will be within 2 standard deviations of the mean of the distribution?

4. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample of size n from a distribution for which the mean is 6.5 and the variance is 4. Determine how large the value of n must be in order for the following relation to be satisfied:
$$\Pr(6 \leqslant \overline{X}_n \leqslant 7) \geqslant 0.8.$$

5. Suppose that 30% of the items in a large manufactured lot are of poor quality. Suppose also that a random sample of n items is to be taken from the lot, and let Q_n denote the proportion of the items in the sample that are or poor quality. Use the Chebyshev inequality to find the value of n such that
$$\Pr(0.2 \leqslant Q_n \leqslant 0.4) \geqslant 0.75.$$