Assignment #6

Due on Friday, September 27, 2013

Read Sections 2.5 and 2.6 on *Independent Events* and *Conditional Probability*, respectively, in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`
Read Section 2.1 on *The Definition of Conditional Probability* in DeGroot and Schervish.
Read Section 2.2 on *Independent Events* in DeGroot and Schervish.
Read Section 2.3 on *Bayes' Theorem* in DeGroot and Schervish.

Do the following problems

1. A machine produces defective parts with three different probabilities depending on its state of repair. If the machine is in good working order, it produces defective parts with probability 0.02. If is wearing down, it produces defective parts with probability 0.1. If it needs maintenance, it produces defective parts with probability 0.3. The probability that the machine is in good working order is 0.8, the probability that it is wearing down is 0.1, and the probability that it needs maintenance is 0.1. Compute the probability that a randomly selected part is defective.

2. Let $E_1, E_2, \ldots E_k$ be independent events with probabilities $p_1, p_2, \ldots, p_k$, respectively. Show that the probability of at least one of $E_1, E_2, \ldots E_k$ occurring is
$$1 - (1 - p_1)(1 - p_2) \cdots (1 - p_k).$$

3. Suppose a fair die is rolled six independent times. A match occurs if side i is observed in the ith trial, $i = 1, 2, \ldots, 6$.
   (a) What is the probability of at least one match in the six rolls?
   (b) Extend the result of part (a) to a fair n–sided die with n independent rolls. What is the limit of the probability as $n \to \infty$?

4. A die is cast independently until a 6 appears. If the casting stops on an odd number of times, Jane wins; otherwise, Jim wins.
   (a) Assume the die is fair. What is the probability that Jane wins?
   (b) Let p denote the probability of a 6. Show that the game favors Jane for all values of p, $0 < p < 1$.

5. A person answers each of two multiple choice questions at random. If there are four possible choices on each question, what is the conditional probability that both answers are correct given that at least one is correct?