

Exam 1

Friday, October 4, 2013

Name: _____

This is a closed book exam. Show all significant work and explain your reasoning. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 4 problems. Relax.

1. Let $(\mathcal{C}, \mathcal{B}, \Pr)$ denote a probability space, and let A, B and C be events in \mathcal{B} .
 - (a) Given that $\Pr(A \cap B) = 0.3$ and $\Pr(A \cap B^c) = 0.1$, compute $\Pr(A)$. Explain your reasoning.
 - (b) Assume A and B are independent with $\Pr(A) = 0.25$ and $\Pr(B) = 0.75$. Compute $\Pr(A \setminus B)$. Explain your reasoning.
 - (c) Define what it means for A, B, C to be mutually independent.
 - (d) Assume that A, B and C are mutually independent with $\Pr(A) = \Pr(B) = \Pr(C) = 1/3$. Compute $\Pr(A \cup B \cup C)$. Explain your reasoning.
2. A box contains four red balls and three blue balls. Suppose that three of the balls are selected at random and without replacement.
 - (a) Compute the probability that all three of the selected balls are blue.
 - (b) Compute the probability that the selected group of three balls contains at least one of the red balls.
3. I have two coins in my pocket; one is a fair coin and the other is a two-headed coin. I grab one of the coins at random and then flip it.
 - (a) Compute the probability that the flip will yield a head.
 - (b) If the flipped coin shows a head, what is the probability that it is the fair coin?
4. A company takes out an insurance policy to cover accidents that occur at its manufacturing plant. Assume that the probability that one or more accidents will occur during any given month is $3/5$, and that the number of accidents that occur in any given month is independent of the number of accidents that occur in all other months. Let X denote the number of months until at least one accident occurs; e.g., $X = 1$ if at least one accident occurs in the first month; $X = 2$ if no accident occurs in the first month, but at least one accident occurs in the second month; etc.
 - (a) Compute the pmf of X .
 - (b) Calculate the probability that there will be at least three consecutive months in which no accidents occur.