Exam 2

Wednesday, November 6, 2013

Name: ____________________________________

This is a closed book exam. Show all significant work and explain your reasoning. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 4 problems. Relax.

1. In each of the following, X and Y denote independent random variables. In each case, set $Z = X + Y$ and compute the mgf, ψ_Z, of Z; then use ψ_Z to determine the distribution of Z.
   (a) $X \sim \text{Binomial}(n, p)$ and $Y \sim \text{Binomial}(m, p)$, where m and n are positive integers and $0 < p < 1$.
   (b) $X \sim \text{Normal}(\mu, 1/\sqrt{2})$ and $Y \sim \text{Normal}(-\mu, 1/\sqrt{2})$, where μ is a real parameter.

2. The moment generating function of a random variable, X, is given by
$$\psi_X(t) = \frac{1}{1-2t}, \qquad \text{for } t < \frac{1}{2}.$$
   (a) Compute $E(X)$ and $\text{Var}(X)$.
   (b) Give the distribution of X and use it to find a value of m for which
$$\Pr(X \leqslant m) = \frac{1}{2}.$$

3. Assume that the joint pdf of a random vector (X, Y) is given by the function
$$f(x,y) = \begin{cases} c(2 - xy^2), & \text{for } 1 \leqslant x \leqslant 2 \text{ and } 0 \leqslant y \leqslant 1; \\ 0, & \text{elsewhere,} \end{cases}$$
   where c is a positive constant.
   (a) Determine the value of c.
   (b) Determine the marginal distribution, f_X, and compute $E(X)$.

4. Let X denote the time a patient spends at a waiting room of a doctor's office waiting to be seen by a physician, and Y the time the physician actually spends with the patient. Assume that X and Y are independent random variables with $X \sim \text{Exponential}(40)$ and $Y \sim \text{Exponetial}(20)$, where X and Y are measured in minutes.
   (a) On average, how long will a patient spend at the waiting room, and how long does the patient spends being seen by a doctor?
   (b) What is the expected value of the time a patient will spend at the doctor's office? Explain your reasoning.
   (c) Give the joint distribution of (X, Y).
   (d) Set up (but DO NOT EVALUATE) the iterated double integral that yields the probability that a patient will spend less than an hour at a doctor's office.