Solutions to Assignment #2

1. *(See Exercise 17 on page 58 in the text)* The distance from Los Angeles to San Francisco is about 382 miles.

   (a) Estimate the distance from Los Angeles to San Francisco in kilometers.

   Solution: The distance from Los Angeles ro San Francisco is about

   $$382 \text{ miles} \doteq 382 \text{ miles} \cdot \frac{1.6 \text{ km}}{1 \text{ mile}} \doteq 611 \text{ km}.$$

   $\square$

   (b) Suppose you drive from Los Angeles to San Francisco at an average speed of 80 kilometers per hour. Estimate the driving time of the trip.

   Solution: If v denotes the speed of the vehicle, then, assuming that the speed is constant,

   $$v = \frac{d}{t},$$

   where d is the distance traveled and t is the travel time. We then have that

   $$t = \frac{d}{v},$$

   where $d = 611$ km from the previous part, and v is given to be 80 km/hr. Thus, the driving time is

   $$t \doteq \frac{611 \text{ km}}{80 \text{ km/hr}} \doteq 7.6375 \text{ hr},$$

   or about 7 hours and 38 minutes. $\square$

2. *(See Exercise 20 on page 58 in the text)* Common speed limits in California are 25 mph, 30 mph, 45 mph, 50 mph , 55 mph, 65 mph and 70 mph. What are these limits in kilometers per hour?

   Solution: Using the information that there are about 1.6 km in one mile, we obtain

   $$25\ \frac{\text{miles}}{\text{hr}} \doteq 25\ \frac{\text{miles}}{\text{hr}} \cdot \frac{1.6 \text{ km}}{1 \text{ mile}} \doteq 40\ \frac{\text{km}}{\text{hr}}.$$

   Table 1 shows the results of the remaining calculations. $\square$

Speed in mph	Speed in km/hr
25	40
30	48
45	72
50	80
55	88
65	104
70	112

Table 1: Results of Calculations in Problem 2

3. *(See Exercise 31 on page 59 in the text)* A football field measures 40 yards by 100 yards. Assume that, at high noon on a very clear day, each square centimeter of field absorbs solar energy at a rate of 1.372×10^{-1} Joules per second. A typical power plant produces energy at a rate of about 100 million Joules per second

   (a) Estimate the area of the football filed in square centimeters.

   ***Solution**:* The dimensions of the football field are: length, $\ell = 100$ yards, and width, $w = 40$ yards. Using the information that there about 91.44 centimeters in a yard, we convert these dimensions to centimeters we get

$$\ell = 100 \text{ yards} \cdot \frac{91.44 \text{ cm}}{1 \text{ yard}} = 9144 \text{ cm}$$

   and

$$w = 40 \text{ yards} \cdot \frac{91.44 \text{ cm}}{1 \text{ yard}} \doteq 3658 \text{ cm}.$$

   Thus, an estimate for the area, A, of the football filed is

$$A = \ell w \doteq 3.34 \times 10^7 \text{ cm}^2.$$

$\square$

   (b) How many Joules of solar energy are absorbed by one football field in one second at high noon.

   ***Solution**:* Given that the rate, R, of energy absorption is about

$$R = 1.372 \times 10^{-1} \ \frac{\text{Joules}}{\text{sec} \cdot \text{cm}^2},$$

it follows that, the amount of energy, E, absorbed by the field in second is about

$$\begin{aligned} E &= RA\cdot(1 \text{ sec}) \\ &\doteq 1.372\times 10^{-1}\ \frac{\text{Joules}}{\text{sec}\cdot\text{cm}^2}\cdot 3.34\times 10^{7}\ \text{cm}^2\cdot 1\ \text{sec} \\ &\doteq 4.58\times 10^{6}\ \text{Joules}. \end{aligned}$$

□

(c) How many football fields are needed to absorb solar energy at high noon at a rate that would match that of a typical power plant.

***Solution**:* A typical power plant produces energy at a rate of about 100 million Joules per second. According to part (b) in this problem, a football field absorbs energy at a rate of about 4.58×10^6 Joules in one second; thus, it takes about

$$\frac{100\times 10^6\ \text{Joules}}{4.58\times 10^6\ \text{Joules}} \doteq 22$$

football fields to absorb the same amount of energy in one second. □

4. *Exercise 32 on page 59 in the text.* There is a rule of thumb for estimating the distance from a lightning flash. Count out loud from the time you see the lightning flash: "One thousand one, one thousand two, one thousand three, ..." until you hear the thunder; this will tell you roughly how many seconds it took for the sound to reach you. You will need to know that the speed of sound in air is about 741.5 miles per hour.

(a) What is the speed of sounds in miles per second?

***Solution**:* Let v denote the speed of sound in the air; then,

$$v \doteq 741.5\ \frac{\text{miles}}{\text{hr}}\cdot\frac{1\ \text{hr}}{60\ \text{min}}\cdot\frac{1\ \text{min}}{60\ \text{sec}} \doteq 0.2\ \frac{\text{miles}}{\text{sec}},$$

or about $\dfrac{1}{5}$ miles per second. □

(b) If it takes 5.5 seconds for you to hear the thunder, how many miles away was the flash?

***Solution**:* Assuming that the speed of sound in air is constant throughout the motions, we have

$$v = \frac{d}{t},$$

where d is the distance traveled, and t is the time of travel. Solving for d in the previous equation we obtain

$$d = vt,$$

which yields the distance traveled by sound in time t. It the case at hand,

$$d \doteq \frac{1}{5}\,\frac{\text{miles}}{\text{sec}} \cdot (5.5 \text{ sec}) = 1.1 \text{ miles}.$$

Thus, the lightning flash was about 1.1 miles away. □

(c) What is the rule of thumb? If the time elapsed until you hear the thunder is t seconds, how far was the flash in miles?

Solution: The rule of thumb is given by the formula for the distance derived in the previous part:

$$d = vt \doteq \frac{1}{5}t = \frac{t}{5} \quad \text{(in miles)},$$

where t is given in seconds. Thus, the rule of thumb is "divide the number of seconds by 5." □

5. *Exercise 41 on page 61 in the text.* The earth has a nearly circular orbit around the sun. The distance from the earth to the sun is around 92.967×10^6 miles, or one Astronomical Unit [AU].

(a) Estimate the total length of the earth's orbit around the sun.

Solution: Assuming a circular orbit, the length of the earth's orbit is approximated by

$$\ell = 2\pi R,$$

where R is the distance from the sun to the earth, or

$$R \doteq 92.967 \times 10^6 \text{ miles}.$$

we then have that the length of the earth's orbit around the sun is about

$$\ell \doteq 2\pi(92.967 \times 10^6) \text{ miles} \doteq 5.841 \times 10^8 \text{ miles}.$$

□

(b) Given that the earth takes about 365.25 days to go around the sun, estimate the earth's speed along its orbit in miles per second.

***Solution**:* Assuming that the earth moves at a constant speed, v, along its orbit, then

$$v = \frac{\ell}{t},$$

where t is the time it takes for the earth to complete one orbit, or

$$\begin{aligned} t &\doteq 365.25 \text{ days} \cdot \frac{24 \text{ hr}}{1 \text{ day}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \\ &\doteq 3.16 \times 10^7 \text{ sec.} \end{aligned}$$

Thus, the speed of the earth along its orbit is about

$$v \doteq \frac{5.841 \times 10^8 \text{ miles}}{3.16 \times 10^7 \text{ sec}} \doteq 18.5 \ \frac{\text{miles}}{\text{sec}}.$$

□

(c) What fraction of the speed of light is the speed of the earth?

***Solution**:* The speed of light is about 186, 282 miles per second. Thus, the earth's speed along its orbit around the sun is about

$$\frac{18.5}{186,282} \quad \text{or about} \quad 0.01\%$$

of the speed of light. □