

Pomona College
Department of Mathematics

Math 29. Advanced Problem Solving
Fall 2013

Problem Set #1

Note: You will be working in groups on these problems during class and in the mentor sessions. Try to make as much progress as possible and keep good notes on how your group arrives at a solution. Outside of class, you will write up solutions to the problems that you have solved in class and in the mentor sessions. Write up solutions neatly in your *Problem Solving Journal*. To be complete and understandable, a solution often needs complete English sentences intertwined with mathematical calculations. Your aim is to make the solution clear to someone who does not know how to do the problem.

1. Benjamin Franklin dropped oil on a lake's surface and noticed that a given amount of oil could not be induced to spread out beyond a certain area. If the number of drops was doubled, then so was the maximum area to which it would spread. His measurement revealed that 0.1 cm^3 of oil spread to a maximum area of 40 m^2 . How thick is such an oil layer? Express your answer in angstroms
(one angstrom, denoted 1 \AA , equals 10^{-10} m).
2. Franklin actually showed that 1 teaspoon of oil would spread to cover about 0.5 acre. Using the fact that $10^4 \text{ m}^2 = 2.47 \text{ acres}$, determine how many cubic centimeters are there in a teaspoon.
3. Determine the number of molecules per cubic centimeter of liquid water by making use of two pieces of information: (a) liquid water has a density of 1 g/cm^3 , and (b) every 18 grams of water contain Avogadro's number (6.02×10^{23}) of H_2O molecules.
4. It has been proposed that dinosaurs became extinct 65 million years ago because Earth was struck by an asteroid. The idea is that dust from the impact was lofted into the upper atmosphere all around the globe, where it lingered for at least several months and blocked the

sunlight reaching Earth's surface. On the dark and cold Earth that temporarily resulted, many forms of life then became extinct. Available evidence suggests that about 20% of the asteroid's mass ended up as dust spread uniformly over the Earth after eventually settling out of the upper atmosphere. This dust amounted to about 0.02 g/cm^2 of the Earth's surface. The asteroid very likely had a density of about 2 g/cm^3 . Using the fact that the Earth's total surface area is about $5.1 \times 10^{14} \text{ m}^2$, estimate the size of the asteroid. Assuming that the shape of the asteroid was spherical, what was the diameter of the asteroid in kilometers? Note the volume of a sphere is $V = (4/3)\pi r^3$.

5. In the asteroid collision described in the previous problem, additional dust would be produced from the earthly material that was blown out by the impact. It has been estimated that the amount of material from the crater zone which would be blasted out is about 60 times the mass of the asteroid, however only about 20% of the material from the crater zone would be lofted to the upper atmosphere and the rest would settle back into the hole to partially refill it. If, after this partial refilling, there remained a cylindrical crater which was 200 kilometers in diameter, what would be its depth? Assume that the density of the earthly material is 2 g/cm^3 .