Assignment #12

Due on Wednesday, October 22, 2014

Read Section 4.3 on *Moments* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 4.4 on *Moments* in DeGroot and Schervish.

Do the following problems

1. Compute the moment generating function, $\psi_X(t)$, of a continuous random variable X with Uniform$(-1, 2)$ distribution. What should $\psi(0)$ be? Give also the second moment and variance of X.

2. Suppose that X is a random variable for which the mgf is as follows:

$$\psi_X(t) = e^{t^2+3t} \quad \text{for } -\infty < t < \infty.$$

   Find the mean and variance of X.

3. Suppose that X is a random variable for which the mgf is as follows:

$$\psi_X(t) = \frac{1}{6}(4 + e^t + e^{-t}) \quad \text{for } -\infty < t < \infty.$$

   Find the probability distribution of X.

4. Let X be a random variable with moment generating function (mgf) ψ_X.

   (a) Let $Y = cX$, where c is a constant. Compute the mgf of Y in terms of ψ_X.

   (b) Let $Y = X + a$, where a is a constant. Compute the mgf of Y in terms of ψ_X.

5. Let X be a random variable with moment generating function (mgf) ψ_X, expected value μ and variance σ^2. Put $Y = \dfrac{X - \mu}{\sigma}$

   (a) Compute the mgf of Y in terms of ψ_X.

   (b) Use the moment generating function found in part (a) to compute $E(Y)$ and var(Y).