

Assignment #17

Due on Monday, November 3, 2014

Read Section 6.1 on *The Normal Distribution* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Section 5.6 on *The Normal Distributions* in DeGroot and Schervish.

Do the following problems

1. Let X and Y be independent $\text{Normal}(0, 1)$ random variables.

Compute $\Pr(X^2 + Y^2 < 1)$.

2. Let $X_1, X_2, X_3, \dots, X_n$ be independent identically distributed $\text{Normal}(0, 1)$ random. Define

$$Y = X_1 + X_2 + \dots + X_n.$$

Use moment generating functions to determine the distribution of Y .

Compute $E(Y)$ and $\text{Var}(Y)$.

3. Let $X_1, X_2, X_3, \dots, X_n$ be independent identically distributed $\text{Normal}(0, 1)$ random. Define

$$\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n}.$$

Use moment generating functions to determine the distribution of \bar{X} .

Compute $E(\bar{X})$ and $\text{Var}(\bar{X})$.

4. Let X denote a nonnegative random variable. Assume that $\ln(X)$ has a standard normal distribution. Compute the pdf of X .
5. Two instruments are used to measure the height, h , of a tower. The error made by the less accurate instrument is normally distributed with mean 0 and standard deviation $0.0056h$. The error made by the more accurate instrument is normally distributed with mean 0 and standard deviation $0.0044h$.

Let X_1 denote the measurement made by the first instrument and X_2 the measurement made by the second instrument. Assume that X_1 and X_2 are independent random variables, and let $X = \frac{X_1 + X_2}{2}$, the average of the two instruments.

- (a) Determine the distribution of X .
- (b) Compute the probability that their average of the two measurements is within $0.005h$ of the height of the tower?