

Assignment #22

Due on Monday, November 24, 2014

Read Section 8 on *Introduction to Estimation* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Section 6.2 on *The Law of Large Numbers* in DeGroot and Schervish.

Do the following problems

1. Let X denote a random variable with mean μ and variance σ^2 . Use Chebyshev's inequality to show that

$$\Pr(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2},$$

for all $k > 0$.

2. Suppose that factory produces a number X of items in a week, where X can be modeled by a random variable with mean 50. Suppose also that the variance for a week's production is known to be 25. What can be said about the probability that this week's production will be between 40 and 60?
3. How large a random sample must be taken from a given distribution in order for the probability to be at least 0.99 that the sample mean will be within 2 standard deviations of the mean of the distribution?
4. Suppose that X_1, X_2, \dots, X_n is a random sample of size n from a distribution for which the mean is 6.5 and the variance is 4. Determine how large the value of n must be in order for the following relation to be satisfied:

$$\Pr(6 \leq \bar{X}_n \leq 7) \geq 0.8.$$

5. Suppose that 30% of the items in a large manufactured lot are of poor quality. Suppose also that a random sample of n items is to be taken from the lot, and let Q_n denote the proportion of the items in the sample that are of poor quality. Use the Chebyshev inequality to find the value of n such that

$$\Pr(0.2 \leq Q_n \leq 0.4) \geq 0.75.$$