Assignment #23

Due on Wednesday, November 26, 2014

Read Section 8 on *Introduction to Estimation* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 6.2 on *The Law of Large Numbers* in DeGroot and Schervish.

Do the following problems

1. Let X be continuous random variable with $E(|X|) < \infty$. Derive the following version of Markov's inequality: For every $\varepsilon > 0$,
$$\Pr(|X| \geqslant \varepsilon) \leqslant \frac{E(|X|)}{\varepsilon}.$$

2. Let X denote a positive random variable with mean 1. Estimate the smallest natural number n such that
$$\Pr(X < n) \geqslant 0.95.$$

3. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a distribution with
$$E(|X_1|) < \infty.$$
Use Markov's inequality to show that, for any $\varepsilon > 0$,
$$\Pr(|\overline{X}_n| \geqslant \varepsilon) \leqslant \frac{E(|X_1|)}{\varepsilon}.$$

4. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a Poisson(1) distribution. Find the smallest natural number n such that
$$\Pr\left(\sum_{k=1}^{n} X_k < n^2\right) \geqslant 0.95.$$

5. Suppose that X is a random variable with mean and variance both equal to 47. What can be said about $\Pr(0 < X < 94)$?