Exam 2 (Part II)

Due on Monday, November 10, 2014

Name: ______________________________

This is the out–of–class portion of Exam 2. There are three questions in this portion of the exam. This is a closed–book and closed–notes exam; you may consult only the "Special Distributions" and the "Normal Distribution Probabilities Table" handouts. You may work on these questions as long as you wish. Show all significant work and give reasons for all your answers.

Students are expected to work individually on these problems. You may not consult with anyone.

Please, write your name on this page and staple it to your solutions. Turn in your solutions at the start of class on **Monday, November 10, 2014.**

I have read and agree to these instructions. Signature: ____________________

1. The random pair (X, Y) has the joint distribution shown in Table 1.

$X\backslash Y$	2	3	4
1	0.1	0	0.1
2	0.3	0.1	0.2
3	0	0.2	0

Table 1: Joint Probability Distribution for X and Y, $p_{(X,Y)}$

   (a) Compute $\Pr(X < Y)$.

   (b) Compute the marginal distributions of X and Y.

   (c) Show that X and Y are not independent. Give a reason for your answer.

   (d) Compute the expectations $E(X)$, $E(Y)$ and $E(XY)$.

   (e) Compute the covariance of X and Y.

2. A random point (X, Y) is distributed uniformly on the triangle with vertices $(0, 0)$, $(4, 0)$ and $(0, 1)$ in the xy–plane.

   (a) Give a formula for computing the joint pdf, $f_{(X,Y)}$, of the random vector (X, Y).

   (b) Compute the marginal distributions f_X and f_Y.

   (c) Are X and Y independent random variables? Give a reason for your answer.

   (d) Compute the expectations $E(X)$, $E(Y)$ and $E(XY)$

   (e) Compute the covariance of X and Y.

3. Suppose that the joint pdf of the random vector (X, Y) is given by

$$f_{(X,Y)}(x, y) = \frac{1}{2\pi}\, e^{-(x^2+y^2)/2}, \qquad \text{for } -\infty < x < \infty \text{ and } -\infty < y < \infty.$$

   (a) Verify that $f_{(X,Y)}$ is indeed a joint pdf.

   (b) Compute the marginal distributions f_X and f_Y.

   (c) Are X and and Y independent? Give a reason for your answer.

   (d) Determine the distribution of $X + Y$.

   (e) Compute $\Pr(-\sqrt{2} < X + Y \leqslant 2\sqrt{2})$.