

Exam 3 (Part II)

Due on Monday, December 8, 2014

Name: _____

This is the out-of-class portion of Exam 3. There are three questions in this portion of the exam. This is a closed-book and closed-notes exam; you may consult only the “Special Distributions” and the “Normal Distribution Probabilities Table” handouts. You may work on these questions as long as you wish. Show all significant work and give reasons for all your answers.

Students are expected to work individually on these problems. You may not consult with anyone.

Please, write your name on this page and staple it to your solutions. Turn in your solutions at the start of class on **Monday, December 8, 2014**.

I have read and agree to these instructions. Signature: _____

1. A company manufactures a brand of incandescent light bulbs. Assume that the light bulbs have a lifetime in months that is normally distributed with mean 3.5 and variance 1; assume also that the lifetimes of the light bulbs are independent. A consumer buys a number of these bulbs with the intention of replacing them successively as they burn out. What is the smallest number of bulbs to be purchased so that the succession of light bulbs produces light for at least 47 months with probability at least 97.5%? Explain your reasoning.
2. Four hundred fair coins are tossed simultaneously. Use the Central Limit Theorem to estimate the probability that exactly 200 of the coins come up heads. Explain your reasoning.
3. Let X_1, X_2, \dots, X_n be a random sample from a Poisson distribution with mean λ , and set $Y_n = \sum_{i=1}^n X_i$, for $n = 1, 2, 3, \dots$

We showed in class that

$$2\sqrt{n} \left(\sqrt{\frac{Y_n}{n}} - \sqrt{\lambda} \right) \xrightarrow{D} Z \sim \text{Normal}(0, 1) \text{ as } n \rightarrow \infty. \quad (1)$$

- (a) Use the fact in (1) to obtain a formula for computing an approximate 95% confidence interval for the mean λ in terms of the sample mean $\bar{X}_n = \frac{Y_n}{n}$.
- (b) Give an approximate 95% confidence interval for λ for a sample size of 36 and a sample mean of 25.