

Assignment #4

Due on Friday, September 19, 2014

Read Section 2.5 on *Subspaces of Euclidean Space* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Section 1.2 on *Subspaces* in Damiano and Little (pp. 12–19)

Background and Definitions

(*Definition of Subspace of \mathbb{R}^n*). A non-empty subset, W , of Euclidean space, \mathbb{R}^n , is said to be a **subspace** of \mathbb{R}^n iff

- (i) $v, w \in W$ implies that $v + w \in W$ (closure under vector addition); and
- (ii) $t \in \mathbb{R}$ and $v \in W$ implies that $tv \in W$ (closure under scalar multiplication).

Do the following problems

1. Let $S = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \mid x \geq 0, y \geq 0 \right\}$. Show that S is closed under vector addition in \mathbb{R}^2 . Explain why S is not a subspace of \mathbb{R}^2 .
2. Let $a_1, a_2, b_1, b_2, c_1, c_2$ be real constants. Let W be the solution set of the homogeneous system
$$\begin{cases} a_1x_1 + b_1x_2 + c_1x_3 = 0 \\ a_2x_1 + b_2x_2 + c_2x_3 = 0. \end{cases}$$
 Prove that W is a subspace of \mathbb{R}^3 .
3. Let $L = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \mid y = 2x + 1 \right\}$. Determine whether or not L is a subspace of \mathbb{R}^2 .
4. Let W be a subspace of \mathbb{R}^n . Use the definition of subspace to prove the following statements.
 - (a) If $v \in W$, then W must also contain the additive inverse of v .
 - (b) W contains the zero vector.
5. Given two subsets A and B of \mathbb{R}^n , the **intersection** of A and B , denoted by $A \cap B$, is the set which contains all vectors that are both in A and B ; in symbols,
$$A \cap B = \{v \in \mathbb{R}^n \mid v \in A \text{ and } v \in B\}.$$
 - (a) Prove that $A \cap B \subseteq A$ and $A \cap B \subseteq B$.
 - (b) Prove that if W_1 and W_2 are two subspaces of \mathbb{R}^n , then the intersection $W_1 \cap W_2$ is a subspace of \mathbb{R}^n which is contained in both W_1 and W_2 .