Exam 1

October 17, 2014

Name: ____________________________________

This is a closed book exam. Show all significant work and explain the reasoning for all your assertions. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 4 problems. Relax.

1. Answer the following questions as thoroughly as possible.

   (a) Let S denote a nonempty subset of $\mathbb{R}^n$. Give a precise definition of span(S).

   (b) Let W denote a subset of $\mathbb{R}^n$. State precisely what it means for W to be a subspace of $\mathbb{R}^n$.

   (c) Let W denote a subspace of $\mathbb{R}^n$. Give the definition of $\dim(W)$.

   (d) State the Fundamental Theorem for Homogeneous Linear Systems.

2. Let S denote a subset of $\mathbb{R}^n$.

   (a) State precisely what it means for S to be linearly independent.

   (b) Let v and w denote vectors in $\mathbb{R}^n$. Prove that if the set $\{v, w\}$ is linearly independent, then the set $\{v, v + w\}$ is linearly independent.

3. Let $W = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \;\middle|\; x - 2y + z = 0 \right\}$

   (a) Explain why W is a subspace of $\mathbb{R}^3$.

   (b) Verify that the set $B = \left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right\}$ is a basis for W.

   (c) Let $v = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. Verify that $v \in W$ and give the coordinates of v relative to B; that is, compute $[v]_B$.

4. Find a basis for the solution space, W, of the homogenous system

$$\left\{ \begin{array}{rcl} x_1 - 2x_3 & = & 0 \\ x_2 + x_3 & = & 0, \end{array} \right.$$

   and compute $\dim(W)$.