Exam 2

December 5, 2014

Name: ______________________________

This is a closed book exam. Show all significant work and explain the reasoning for all your assertions. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 4 problems. Relax.

1. Complete the following definitions:

   (a) A function $f\colon \mathbb{R}^n \to \mathbb{R}^m$ is linear iff ...

   (b) A scalar, λ, is an eigenvalue of a linear transformation $T\colon \mathbb{R}^n \to \mathbb{R}^n$ iff ...

   (c) The null space, $\mathcal{N}_T$, of a linear transformation $T\colon \mathbb{R}^n \to \mathbb{R}^m$ is defined to be ...

2. Let $\mathcal{B} = \{v_1, v_2\}$ be made up of the vectors

$$v_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \text{and} \quad v_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix};$$

   let $\mathcal{B}' = \{w_1, w_2\}$ be made up of the vectors

$$w_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{and} \quad w_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix};$$

   and $\mathcal{E} = \{e_1, e_2\}$ be the standard basis in $\mathbb{R}^2$.

   Let $id\colon \mathbb{R}^2 \to \mathbb{R}^2$ denote the identity map in $\mathbb{R}^2$.

   (a) Compute the change of basis matrices $[id]_{\mathcal{B}}^{\mathcal{E}}$ and $[id]_{\mathcal{B}'}^{\mathcal{E}}$.

   (b) Use your results from part (a) to compute the change of basis matrices $[id]_{\mathcal{E}}^{\mathcal{B}}$ and $[id]_{\mathcal{E}}^{\mathcal{B}'}$.

   (c) Use your results from parts (a) and (b) to compute the change of basis matrix $[id]_{\mathcal{B}}^{\mathcal{B}'}$.

3. Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation satisfying

$$T(e_1) = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \quad T(e_2) = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad \text{and} \quad T(e_3) = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix},$$

where $\{e_1, e_2, e_3\}$ is the standard basis in $\mathbb{R}^3$.

   (a) Give the matrix representation of T relative to the standard basis in $\mathbb{R}^3$.

   (b) Given that $\lambda = 1$ is an eivenvalue of the transformation T, compute the eigenspace, $E_T(1)$, corresponding to this eigenvalue. What is $\dim(E_T(1))$?

4. Let u_1 and u_2 denote a unit vector in $\mathbb{R}^3$ that are orthogonal to each other; i.e., $\langle u_1, u_2\rangle = 0$, where $\langle \cdot, \cdot \rangle$ denotes the Euclidean inner product in $\mathbb{R}^3$.

   (a) Define $f\colon \mathbb{R}^3 \to \mathbb{R}^3$ by $f(v) = \langle v, u_1\rangle u_1 + \langle v, u_2\rangle u_2$ for all $v \in \mathbb{R}^3$. Verify that f is linear.

   (b) Verify that the set $\mathcal{B} = \{u_1, u_2\}$ is a basis for the image, $\mathcal{I}_f$, of f.