Topics for Final Exam

1. **Vector Space Structure in Euclidean Space**

   1.1 Definition of n–Dimensional Euclidean Space

   1.2 Vector addition and scalar multiplication

   1.3 Spans

   1.4 Linear independence

   1.5 Subspaces of Euclidean Space

   1.6 Bases

   1.7 Dimension

   1.8 Ordered bases and coordinates

2. **Connections with the Theory of Linear Equations**

   2.1 Homogeneous systems

   2.2 Fundamental theorem for homogenous systems of linear equations

   2.3 Nonhomogeneous systems

3. **Euclidean Inner Product and Norm**

   3.1 Row–column product and the Euclidean inner product

   3.2 Euclidean norm

   3.3 Orthogonality

4. **Matrices**

   4.1 The set, $\mathbb{M}(m, n)$, of $m \times n$ matrices as a linear space

   4.2 Matrix algebra

   4.3 Null space, column space and row space of a matrix

   4.4 Invertible matrices

5. **Linear Transformations**
   - 5.1 Definition of linearity
   - 5.2 Matrix representation
   - 5.3 Null space; image; the Dimension Theorem
   - 5.4 Compositions
   - 5.5 One–to–one, onto, and invertible linear transformations
   - 5.6 Application: Change of basis matrix item Application: Geometric transformations in the plane (rotations and reflections)

6. **The Eigenvalue Problem**
   - 6.1 Eigenvalues, eigenvectors and eigenspaces
   - 6.2 The eigenvalue problem
   - 6.3 Application: Diagonalization of 2×2 matirces

Relevant sections in text: 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 4.1 and 4.2

Relevant sections in the online class notes: 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 2.10 and 2.11. 2.12, 3.1, 3.2, 3.3, 3.4, 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 5.1 and 5.2

Important Concepts: Euclidean space; linear independence; span; subspaces; bases; dimension; coordinates; inner product; norm; orthogonality; linear transformation; matrix representation of a linear transformation; change of basis matrix; one–to–one functions; onto functions; compositions of functions; invertible functions; null space; image; invertible matrices; eigenvalue, eigenvector and eigenspace of linear transformations.

Important Skills: Know how to determine whether subsets of $\mathbf{R}^n$ are linearly independent; know how to tell whether a given subset of $\mathbf{R}^n$ is a subspace; know how to tell whether a set of vectors in $\mathbf{R}^n$ spans a subspace; know how to compute the span of a set of vectors; know how to solve systems of linear equations; know how to determine bases for subspaces of Euclidean space; know how to compute dimensions of subspaces; know how to find coordinates of vectors relative to ordered bases; know how to tell whether vectors are orthogonal; know how to tell whether a given matrix is invertible or not; know how to compute inverses of invertible matrices; know how to determine whether a given function is linear or not; know how to obtain matrix representations of linear transformations; know how to compute change of basis matrices; know how to compute determinants of 2×2; know how to find eigenvalues, eigenvectors and eigenspaces of linear transformations.