Assignment #5

Due on Thursday, October 13, 2016

Read Section 5.1 on *Expected Value of a Random Variable* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 4.1 on *The Expectation of a Random Variable* in DeGroot and Schervish.

Do the following problems.

1. Let $X \sim \text{Uniform}(a, b)$. Compute $E(X)$.

2. Let X be a continuous random variable with pdf
$$f_X(x) = \frac{1}{\pi(x^2+1)} \quad \text{where } x \in \mathbb{R}.$$
Show that X has no expectation.

3. Suppose that X is a **bounded** and continuous random variable; that is, there exists a positive number M such that
$$\Pr(|X| \leqslant M) = 1.$$
Show that $E(X)$ exists. In other words, show that
$$\int_{-\infty}^{\infty} |x| f_X(x) \, \mathrm{d}x < \infty.$$

4. Suppose a random variable X has a uniform distribution on the interval $[0, 1]$. Show that the expectation of $1/X$ does not exist.

5. Suppose that a point is chosen at random on a stick of unit length at that the stick is broken into two pieces at that point. Find the expected value of the length of the longer piece.

6. Two discrete random variable, X and Y, are said to be **independent** if

$$\Pr(X = x,\ Y = y) = \Pr(X = x) \cdot \Pr(Y = y)$$

for all possible values of x and y or X and Y, respectively.

Prove that if X and Y are discrete and independent, then

$$E(X + Y) = E(X) + E(Y).$$

7. Let X be a discrete random variable with pmf $p_X(x)$, and assume that $p_X(x)$ is positive at $x = -1, 0, 1$ and zero elsewhere.

   (a) If $p_X(0) = \dfrac{1}{4}$, find $E(X^2)$.

   (b) If $p_X(0) = \dfrac{1}{4}$ and if $E(X) = \dfrac{1}{4}$, determine $p_X(-1)$ and $p_X(1)$.

8. A bowl contains 10 chips, of which eight are marked \$2 and two are marked \$5 each. Let a person choose, at random and without replacement, three chips from the bowl. Assume the person is to receive the sum of the resulting amounts. On average, how much money will the person receive?

9. Let $p_X(k) = \left(\dfrac{1}{2}\right)^k$, for $k = 1, 2, 3, \ldots$, zero elsewhere, be the pmf of a discrete random variable X. Find the mean value of X.

   Hint: For $|t| < 1$, define the function $f(t) = \displaystyle\sum_{k=0}^{\infty} t^k$. This is a geometric series which adds up to $\dfrac{1}{1-t}$. Compute $f'(t)$.

10. An experiment consists of tossing a balanced die until a 6 comes up. On average, how many tosses are required to get a 6? In other words, if X denotes the number of tosses it takes to get a 6, what is $E(X)$? Show your calculations and justify your reasoning.