Exam 1

Thursday, September 29, 2016

Name: ________________________________

This is a closed–book and closed–notes exam. Show all significant work and give reasons for all your answers. Use your own paper and/or the paper provided by the instructor. You have up to 75 minutes to work on the following 5 problems. Relax.

1. Let $(\mathcal{C}, \mathcal{B}, \Pr)$ denote a probability space.

   (a) State precisely what it means for $E_1, E_2, E_3 \in \mathcal{B}$ to be mutually exclusive.

   (b) State precisely what it means for $E_1, E_2, E_3 \in \mathcal{B}$ to be mutually independent.

2. Let $(\mathcal{C}, \mathcal{B}, \Pr)$ denote a probability space, and let A and B be elements in the σ–Field $\mathcal{B}$. Recall that $A \setminus B = \{x \in A \mid x \notin B\}$. Use the additivity property of probability to derive the fact: $\Pr(A \setminus B) = \Pr(A) - \Pr(A \cap B)$.

   Provide reasons for the steps in your derivation.

3. Consider a fair six–sided die where one side is labeled 1, and the others are all labeled 0. The die is rolled twice in a row. Let A denote the event that the first die is a 1, B the event that the second die is a 0.

   (a) Give the elements in the sample space for this experiment.

   (b) Give the probability, Pr, associated with each of the sample points.

   (c) Compute $\Pr(A)$ and $\Pr(B)$.

4. For the probability space $(\mathcal{C}, \mathcal{B}, \Pr)$ and events A and B defined in Problem 3,

   (a) compute $\Pr(A \cap B)$ and $\Pr(A \mid B)$.

   (b) Are the events A and B independent? Explain your answer.

5. A box contains two coins: one of the coins is a fair coin and the other one is a two–headed coin.

   (a) You pick a coin at random and toss it. What is the probability that it lands heads up?

   (b) You pick a coin at random and toss it, and get heads. What is the probability that it is the two–headed coin?