Assignment #6

Due on Wednesday, September 28, 2016

Section 3.2 on *Analysis of the Traffic Flow Equation* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`.

Background and Definitions

A Simple Age Structured Population Model. Postulate a population density, $n(a,t)$, which also gives the age distribution for individuals in the population; so that, the number of individuals in the population between the ages a_1 and a_2 at time t is given by $\displaystyle\int_{a_1}^{a_2} n(a,t)\,da$.

Do the following problems.

1. Explain why $n(a,t)$ is given in units of population divided by units of time.

2. Give an expression for the total number of individuals, $N(a,t)$, in the population at time t having ages less than or equal to a.

3. Assuming that a is a differentiable function of t with continuous derivative, use Chain Rule to compute the rate of change of population density at time t, $\dfrac{dn}{dt}$.

   Explain why
   $$\frac{dn}{dt} = \frac{\partial n}{\partial t} + \frac{\partial n}{\partial a}.$$

4. Assume that the death rate for individuals of age a in the population is proportional to the number of individuals at that age with constant of proportionality $\mu(a)$.

   Use a conservation principle to derive the following partial differential equation
   $$\frac{\partial n}{\partial t} + \frac{\partial n}{\partial a} = -\mu(a)n.$$

   Give the characteristic curves for the equation.

5. Give solutions to the partial differential equation derived in the previous problem assuming that the death rate is zero for all ages. Interpret your result.