Assignment #4

Due on Friday, September 23, 2016

Read Section 4.1, *Recovering a Function from its Rate of Change,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 4.2, *The Natural Logarithm Function,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 5.3 on *The Definite Integral,* pp. 369–376, in *Calculus for the Life Sciences* by Schreiber, Smith and Getz.

Read Section 5.4 on *The Fundamental Theorem of Calculus,* pp. 378–384, in *Calculus for the Life Sciences* by Schreiber, Smith and Getz.

Read Section 5.5 on *Substitution,* pp. 386–392, in *Calculus for the Life Sciences* by Schreiber, Smith and Getz.

Do the following problems

1. Solve the initial value problem

$$\begin{cases} \dfrac{dy}{dt} = t\sin(t^2); \\[2ex] y(0) = 0, \end{cases}$$
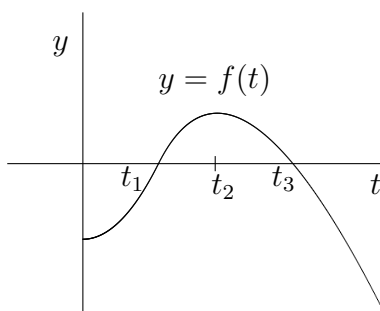
   for $t \in \mathbf{R}$.

2. Solve the initial value problem

$$\begin{cases} \dfrac{dy}{dt} = \dfrac{\cos(\pi + \sqrt{t})}{\sqrt{t}}; \\[2ex] y(\pi^2) = 1, \end{cases}$$

   for $t \geqslant 0$.

3. Let the graph of $y = f(t)$ be as sketched in Figure 1 on page 2 and put

$$F(t) = \int_0^t f(\tau) \; d\tau, \text{ for } t \geqslant 0.$$

Figure 1: Sketch of graph of $y = f(t)$

(a) Based on the sketch in Figure 1, determine intervals on which (i) $F(t)$ increases, (ii) $F(t)$ decreases, (iii) the graph of $y = F(t)$ is concave up, and (iv) the graph of $y = F(t)$ is concave down.

(b) Estimate the times at which $F(t)$ is (i) a local maximum, and (ii) (i) a local minimum.

(c) Locate any inflection points in the graph of $y = F(t)$

4. Let f and F be as in Problem 3. Use the qualitative information obtained in Problem 3 to sketch the graph of $y = F(t)$.

5. Let f and F be as in Problem 3. Given that $t_2 = 2$ and t_3 is about 3 and $f(t_2)$ is about 0.75, estimate the maximum value of F over the range of values of t pictured in Figure 1.

Suggestion: For this problem you may also assume the estimates $f(0) \approx -1$ and $t_1 \approx 1$.