Assignment #5

Due on Monday, September 26, 2016

Read Section 4.2, *The Natural Logarithm Function,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 5.5 on *Substitution,* pp. 386–392, in *Calculus for the Life Sciences* by Schreiber, Smith and Getz.

Background and Definitions

The natural logarithm function, $\ln\colon (0,\infty)\to\mathbf{R}$, is the unique solution to the initial value problem

$$\begin{cases} \dfrac{dy}{dt} = \dfrac{1}{t}; \\ y(1) = 0, \end{cases}$$

for $t > 0$, so that

$$\ln(t) = \int_1^t \frac{1}{\tau}\, d\tau, \qquad \text{for all } t > 0.$$

Using this definition, we derived the follow properties of the natural logarithm function in class.

(i) $\ln(1) = 0$;

(ii) $\ln\colon (0,\infty)\to\mathbf{R}$ is differentiable and $\ln'(t) = \dfrac{1}{t}$, for all $t > 0$;

(iii) $\ln(ab) = \ln a + \ln b$ for all $a, b > 0$;

(iv) $\ln(b^p) = p\ \ln b$ for all $b > 0$ and $p \in \mathbf{R}$.

Do the following problems

1. Derive the following additional properties of the natural logarithm function.

   (a) $\ln\left(\dfrac{1}{b}\right) = -\ln b$, for $b > 0$.

   (b) $\ln\left(\dfrac{a}{b}\right) = \ln a - \ln b$, for $a, b > 0$.

2. Let $f(t) = \ln\sqrt{1+t^2}$ for all $t \in \mathbf{R}$.

   (a) Compute $f'(t)$ and $f''(t)$.

   (b) Determine the intervals on the t–axis for which f is increasing or decreasing, and all local extrema; the values of t for which the graph of $y = f(t)$ is concave up, and those for which the graph is concave down; and all the inflection points of the graph of $y = f(t)$.

   (c) Using the information in the previous part, sketch the graph of $y = f(t)$.

3. Let $f(t) = t\ln t$ for $t > 0$.

   (a) Compute $f'(t)$ and $f''(t)$.

   (b) Determine the intervals on the t–axis for which f is increasing or decreasing, and all local extrema; the values of t for which the graph of $y = f(t)$ is concave up, and those for which the graph is concave down; and all the inflection points of the graph of $y = f(t)$.

   For this problem, you will need the fact that $\ln e = 1$.

   (c) Using the limit facts

   $$\lim_{t\to 0^+} t\ln t = 0 \qquad \text{and} \qquad \lim_{t\to\infty} t\ln t = \infty,$$

   and the information in the previous part, sketch the graph of $y = f(t)$. Sketch the graph of $y = f(t)$.

4. Evaluate the indefinite integral

   $$\int \frac{1}{t+\sqrt{t}}\, dt$$

   by making the change of variables $u = \sqrt{t}$.

5. Define $g(t) = t\ln t - t$ for all $t > 0$. Compute $g'(t)$ and use your result in order to obtain a formula for evaluating the indefinite integral

   $$\int \ln u\, du.$$