Topics for Exam 1

1. **Introduction to Modeling**

    1.1 Conservation principles: One–Compartment Models

    1.2 Models of population growth

2. **Applications of Differential Calculus, Part I**

    2.1 Qualitative study of the first order differential equation: $\dfrac{dy}{dt} = g(y)$.

    2.2 Application: Qualitative analysis of models of population growth.

3. **Applications of Integral Calculus**

    3.1 Applications of The Fundamental Theorem of Calculus

      1.1 Recovering a function from its rate of change.

      1.2 Solving the initial value problem $\begin{cases} \dfrac{dy}{dt} = f(t); \\ y(t_o) = y_o, \end{cases}$ where f is a continuous function defined on an interval containing t_o.

      1.3 Evaluating integrals: Changing variables

    3.2 The natural logarithm and exponential functions

      3.1 Definition

      3.2 Properties

Relevant chapters and sections in the online class notes: Chapter 2; Sections 3.1, 3.2, 4.1, 4.2, 4.3 and 4.4

Relevant sections in the textbook: Sections 5.4, 5.3, 5.5, 1.6 and 1.4

Important Concepts: Conservation principle, rates, differential equation, initial value problem.

Important Results:

A conservation principle for a one-compartment model. Let $Q(t)$ denote the amount of a substance in a compartment at time t. Then, the rate of the change of the substance in the compartment is determined by the differential equation:

$$\frac{dQ}{dt} = \text{Rate of } Q \text{ in} - \text{Rate of } Q \text{ out},$$

where we are assuming that Q is a differentiable function of time.

Recovering a function from its rate of change. Let I denote an open interval of real numbers and $t_o \in I$. Let $f\colon I \to \mathbf{R}$ be a continuous real–valued function and $y_o \in \mathbf{R}$. The unique solution of the initial value problem

$$\begin{cases} \dfrac{dy}{dt} = f(t) \\ y(t_o) = y_o. \end{cases}$$

is given be the function $y\colon I \to \mathbf{R}$ defined by

$$y(t) = y_o + \int_{t_o}^{t} f(\tau)\, d\tau, \qquad \text{for all } t \in I.$$

The natural logarithm function, $\ln\colon (0,\infty) \to \mathbf{R}$, is the unique solution to the initial value problem

$$\begin{cases} \dfrac{dy}{dt} = \dfrac{1}{t}; \\ y(1) = 0, \end{cases}$$

for $t > 0$; so that,

$$\ln(t) = \int_{1}^{t} \frac{1}{\tau}\, d\tau, \qquad \text{for all } t > 0.$$

The exponential function, $\exp\colon \mathbf{R} \to (0,\infty)$, given by $\exp(t) = e^t$, for all $t \in \mathbf{R}$, is the unique solution of the initial value problem

$$\begin{cases} \dfrac{dy}{dt} = y; \\ y(0) = 1. \end{cases}$$

Important Skills:

- Know how to apply conservation principles to derive differential equation models.
- Know how to apply the Fundamental Theorem of Calculus to obtain the solutions the initial value problem $\begin{cases} \dfrac{dy}{dt} = f(t); \\ y(t_o) = y_o, \end{cases}$ where f is a continuous function defined on an interval containing t_o.
- Know how to obtain qualitative information about solutions of first order differential equations.
- Know how to use the properties of the natural logarithm and exponential functions.
- Know how to use change of variables to evaluate indefinite integrals.