Topics for Final Exam

1. **Introduction to Mathematical Modelling**

   1.1 Conservation principles: One–Compartment Models

   1.2 Models of population growth

      2.1 Malthusian model

      2.2 Logistic model

2. **Qualitative Analysis, Part I**

   2.1 Qualitative study of the first order differential equation: $\dfrac{dy}{dt} = g(y)$.

   2.2 Application: Qualitative analysis of models of population growth.

3. **Applications of Integral Calculus**

   3.1 Applications of The Fundamental Theorem of Calculus

      1.1 Recovering a function from its rate of change.

      1.2 Solving the initial value problem $\begin{cases} \dfrac{dy}{dt} = f(t); \\ y(t_o) = y_o, \end{cases}$ where f is a continuous function defined on an interval containing t_o.

      1.3 Evaluating integrals: Changing variables

   3.2 The natural logarithm and exponential functions

      3.1 Definition

      3.2 Properties

4. **Solving First Order Differential Equations**

   4.1 Separation of variables.

   4.2 Solving first order linear differential equations by the method of integrating factors.

   4.3 Additional techniques of integration:

      3.1 Integration by parts.

      3.2 Partial fractions

5. **Linear Approximations**

   5.1 Linear approximation to a differentiable function.

   5.2 Error in the linear approximation.

6. **Qualitative Analysis, Part II**

   6.1 Isolated equilibrium points; stability and asymptotic stability; unstable equilibrium point.

   6.2 Existence and uniqueness; global existence and long–term behavior.

   6.3 Principle of Linearized Stability

Relevant chapters in the class lecture notes: 2, 3, 4 and 5.

Relevant sections in the textbook: 1.4, 1.6, 5.1, 5.3, 5.4, 5.5, 5.6, 6.1 and 6.2.

Relevant Assignments: 1–18.

Important Concepts: Differential equation, initial value problem, conservation principle, linear approximation, linearized equation, equilibrium point, stability.

Important Skills:

- Know how to apply conservation principles to derive differential equation models.
- Know how to apply the Fundamental Theorem of Calculus to obtain the solutions the initial value problem $\begin{cases} \dfrac{dy}{dt} = f(t); \\ y(t_o) = y_o, \end{cases}$ where f is a continuous function defined on an interval containing t_o.
- Know how to obtain qualitative information about solutions of first order differential equations.
- Know how to use the properties of the natural logarithm and exponential functions.
- Know how to use change of variables to evaluate indefinite integrals.
- know how to use separation of variables to solve first order differential equations.
- Know how to use integration by parts to evaluate integrals.
- Know how to use partial fractions to evaluate integrals.
- Know how to compute linear approximations to differentiable functions.
- Know how to estimate the error in the linear approximation.
- Know how to apply the principle of linearized stability.