## Review Problems for Exam 1

(1) There are 5 red chips and 3 blue chips in a bowl. The red chips are numbered $1, 2, 3, 4, 5$ respectively, and the blue chips are numbered $1, 2, 3$ respectively. If two chips are to be drawn at random and without replacement, compute the probability that these chips are have either the same number or the same color.

(2) A person has purchased 10 of 1,000 tickets sold in a certain raffle. to determine the five prize winners, 5 tickets are drawn at random and without replacement. Compute the probability that this person will win at least one prize.

(3) Let $(\mathcal{C}, \mathcal{B}, \Pr)$ denote a probability space, and let $E_1$, $E_2$ and $E_3$ be mutually disjoint events in $\mathcal{B}$. Find $\Pr[(E_1 \cup E_2) \cap E_3]$ and $\Pr(E_1^c \cup E_2^c)$.

(4) Let $(\mathcal{C}, \mathcal{B}, \Pr)$ denote a probability space, and let $A$ and $B$ events in $\mathcal{B}$. Show that

$$\Pr(A \cap B) \leq \Pr(A) \leq \Pr(A \cup B) \leq \Pr(A) + \Pr(B).$$

(5) Let $(\mathcal{C}, \mathcal{B}, \Pr)$ denote a probability space, and let $E_1$, $E_2$ and $E_3$ be mutually independent events in $\mathcal{B}$ with probabilities $\dfrac{1}{2}$, $\dfrac{1}{3}$ and $\dfrac{1}{4}$, respectively. Compute the exact value of $\Pr(E_1 \cup E_2 \cup E_3)$.

(6) Let $(\mathcal{C}, \mathcal{B}, \Pr)$ denote a probability space, and let $E_1$, $E_2$ and $E_3$ be mutually independent events in $\mathcal{B}$ with $\Pr(E_1) = \Pr(E_2) = \Pr(E_3) = 0.25$. Compute $\Pr[(E_1^c \cap E_2^c) \cup E_3]$.

(7) A bowl contains 5 chips of the same size and shape. One the chips is red and the rest are blue. Draw chips from the bowl at random, one at a time and without replacement, until the red chip is drawn.

   (a) Describe the sample space of this experiment.

   (b) Define the probability function for this experiment. Justify your answer.

   (c) Compute the probability that at least two draws will be needed to get the red chip.

(8) Dreamboat cars are produced at three different factories A, B and C. Factory A produces 20 percent of the total output of Dreamboats, B produces 50 percent, and C produces 30 percent. However, 5 percent of the cars produced at A are lemons, 2 percent of those produced at B are lemons, and 10 percent of those produced at C are lemons. If you buy a Dreamboat and it turns out to be lemon, what is the probability that it was produced at factory A?

(9) Let $(\mathcal{C}, \mathcal{B}, \Pr)$ denote a probability space, and let $A$ and $B$ events in $\mathcal{B}$. Given that $\Pr(A) = 1/3$, $\Pr(B) = 1/5$ and $\Pr(A \mid B) + \Pr(B \mid A) = 2/3$, compute $\Pr(A^c \cup B^c)$.

(10) Let $(\mathcal{C}, \mathcal{B}, \Pr)$ denote a probability space, and let $A$ and $B$ independent events in $\mathcal{B}$ with $\Pr(B) > 0$. Given that $\Pr(A) = 1/3$, compute $\Pr(A \cup B^c \mid B)$.