Assignment #4

Due on Monday, September 24, 2018

Read Section 1.2 on *The Vector Space* $\mathbb{R}^n$ in Baxandall and Liebek's text (pp. 2–9).

ReadSection 2.5 on *The Cross Product* in the class Lecture Notes at
`http://pages.pomona.edu/~ajr04747/`.

Do the following problems

1. Let u, v and w denote non–zero vectors in $\mathbb{R}^3$. Given that $u \cdot w = 0$, $u \cdot v = c$, where c is a real constant, and $u \times v = w$, find the components of v in each of the three mutually orthogonal directions: u, w and $u \times w$.

2. Prove that the cross product is non–associative; that is, find three vectors u, v and w in $\mathbb{R}^3$ such that $(u \times v) \times w \neq u \times (v \times w)$.

3. Let v and w denote vectors in $\mathbb{R}^3$, and $\mathbf{0}$ the zero–vector in $\mathbb{R}^3$.

   (a) Prove that if $v \times w = \mathbf{0}$ and $v \cdot w = 0$, then at least one of v or w must be the zero vector.

   (b) Prove that $v \cdot (v \times w) = 0$.

4. In this problem and the next, we derive the vector identity
$$u \times (v \times w) = (u \cdot w)v - (u \cdot v)w$$
for any vectors u, v and w in $\mathbb{R}^3$.

   (a) Argue that $u \times (v \times w)$ lies in the span of v and w. Consequently, there exist scalars t and s such that
$$u \times (v \times w) = tv + sw$$

   (b) Show that $(u \cdot v)t + (u \cdot w)s = 0$.

5. Let u, v and w be as in the previous problem.

   (a) Use the results of the previous problem to conclude that there exists a scalar r such that
$$u \times (v \times w) = r[(u \cdot w)v - (u \cdot v)w].$$

   (b) By considering some simple examples, deduce that $r = 1$ in the previous identity