

Topics for Final Exam**1. Euclidean Space**

- 1.1 Definition of n -Dimensional Euclidean Space
- 1.2 Spans, lines and planes
- 1.3 Dot product and Euclidean norm
- 1.4 Orthogonality and projections
- 1.5 The cross product in \mathbf{R}^3
- 1.6 The triple scalar product in \mathbf{R}^3

2. Continuous Functions

- 2.1 Vector fields, scalar fields and paths
- 2.2 Definition of continuous function
- 2.3 Compositions of continuous functions
- 2.4 Limits and continuity

3. Differentiability

- 3.1 Definition of differentiability
- 3.2 The derivative as a linear approximation
- 3.3 Derivatives of vector valued functions
- 3.4 Derivatives of scalar fields
 - i. The gradient
 - ii. Partial derivatives
 - iii. Directional derivatives
- 3.5 The Jacobian matrix of a differentiable function
- 3.6 The derivative of a composition of functions: The Chain Rule

4. Differentiable Paths

- 4.1 C^1 curves and parametrizations
- 4.2 Simple C^1 curves.
- 4.3 Piecewise C^1 simple curves
- 4.4 Simple closed curves
- 4.5 Arclength of a curve; arclength parameter

5. Integrals on Curves

- 5.1 Re-parametrizations
- 5.2 Path integrals
- 5.3 Line integrals
- 5.4 Flux across a closed curve

6. Differential Forms and the Fundamental Theorem of Calculus

- 6.1 Differential 1-forms
- 6.2 Differential 0-forms
- 6.3 The differential of a 0-form
- 6.4 Differential 2-forms
- 6.5 Wedge product of 1-forms
- 6.6 Algebra of forms
- 6.7 The differential of a 1-form
- 6.8 Calculus of forms
- 6.9 Integration of forms
- 6.10 The Fundamental Theorem of Calculus
- 6.11 Evaluating differential 2-forms in oriented triangles: Double integrals

Relevant sections in the text: Sections 1.2, 2.1, 2.3, 2.5, 2.6, 2.7, 3.3, 3.6, 3.7, 3.8, 4.1, 4.2, 4.3, 4.4, 5.2, 5.3, 5.4, 5.5, 11.2 and 11.3.

Relevant chapters in the online class notes: Chapters 2, 3, 4 and 5.

Important Concepts: Euclidean space, dot product, orthogonal projections, cross product, continuous function, differentiability, the derivative map, partial derivatives, the gradient of a scalar field, C^1 curves, piecewise C^1 curves, simple curves, simple closed curves, parametrizations, re-parametrizations, arclength, path integral, line integral, flux, differential forms, wedge product of forms, and double integrals.

Important Skills: Know how to compute projections; know how to find equations of lines and planes; know how to show whether a function is continuous or not; know how to show whether a function is differentiable or not; know how to compute partial derivatives, gradients and directional derivative of scalar fields; Know how to compute the Jacobian matrix of a differentiable map; know how to apply the Chain Rule; know how to evaluate the arclength of C^1 curves; know how to evaluate path integrals; know how to evaluate line integrals; know how to compute flux across a simple closed curve; know how to evaluate differential 1-forms and differential 2-forms; know how to evaluate double integrals; know how to apply the Fundamental Theorem of Calculus.