Assignment #12

Due on Monday March 10, 2008

Read Section 4.1 on *The Expectation of a Random Variable,* pp. 181–188, in DeGroot and Schervish.

Read Section 4.2 on *Properties of Expectations,* pp. 189–196, in DeGroot and Schervish.

Do the following problems

1. A balanced die is tossed n times. Let X denote the number of 1's that come up. Give the pmf for X and compute its expectation.

2. Let X and Y denote independent Binomial(n,p) random variables and put $Z = X + Y$. Determine the pmf of Z and compute its expectation.

   Hint: Suppose there are n red balls and n blue balls in a box. Compute the number of ways of picking k balls out of the box, l of which are red and $k - l$ of which are blue.

3. *(Random Walk on the Integers).* A particle starts at $x = 0$ and, after one unit of time, it moves one unit to the right with probability p, for $0 < p < 1$, or to the left with probability $1 - p$. Let X_1 denote the position of the particle after one unit of time and X_2 denote that after 2 units of time. Give the probability mass functions for X_1 and X_2 and compute their expectations. Assume that at each time step, whether a particle will move to the right or to the left is independent of where it has been.

4. *(Random Walk on the Integers, Continued).* Let X_3 denote the position of the particle in the previous problem after 3 units of time. Give its pmf and expectation. Generalize this result to X_n, the position of the particle after n units of time.

5. Toss a coin 100 times, and let X denote the number of heads that come up. Given that the probability of a head is p, where $0 < p < 1$, give the distribution function of X and compute $\Pr(35 \leqslant X \leqslant 45)$ for the cases $p = 0.5$ and $p = 0.4$.