Assignment #3

Due on Monday February 4, 2008

Read Section 1.5 on *The Definition of Probability,* pp. 12–18, in DeGroot and Schervish.

Do the following problems

1. Exercise 3 on page 18 in the text

2. Exercise 7 on page 18 in the text

3. Exercise 9 on page 18 in the text

4. Let A and B be elements in a σ–field $\mathcal{B}$ on a sample space $\mathcal{C}$, and let Pr denote a probability function defined on $\mathcal{B}$. Recall that $A \setminus B = \{x \in A \mid x \notin B\}$. Prove that if $B \subseteq A$, then
$$\Pr(A \setminus B) = \Pr(A) - \Pr(B).$$

5. Let $(\mathcal{C}, \mathcal{B}, \Pr)$ denote a probability space, and B an event in $\mathcal{B}$ with $\Pr(B) > 0$. Let
$$\mathcal{B}_B = \{D \subset \mathcal{C} \mid D = E \cap B \text{ for some } E \in \mathcal{B}\}.$$
We have already seen that $\mathcal{B}_B$ is a σ–field.

   Let $P_{_B} : \mathcal{B}_B \to \mathbb{R}$ be defined by $P_{_B}(A) = \dfrac{\Pr(A)}{P(B)}$ for all $A \in \mathcal{B}_B$. Verify that $(B, \mathcal{B}_B, P_{_b})$ is a probability space; that is show that $P_{_B} : \mathcal{B}_B \to \mathbb{R}$ is a probability function.