Assignment #4

Due on Wednesday February 6, 2008

Read Section 1.5 on The Definition of Probability, pp. 12–18, in DeGroot and Schervish.
Read Section 1.6 on Finite Sample Spaces, pp. 19–22, in DeGroot and Schervish.

Do the following problems

1. Let \((C, \mathcal{B}, \Pr)\) be a sample space. Suppose that \(E_1, E_2, E_3, \ldots\) is a sequence of events in \(\mathcal{B}\) satisfying
   \[E_1 \supseteq E_2 \supseteq E_3 \supseteq \cdots.\]
   Then, \(\lim_{n \to \infty} \Pr(E_n) = \Pr(\bigcap_{k=1}^{\infty} E_k).\)
   
   *Hint:* Use the analogous result for an increasing nested sequence of events presented in class and De Morgan’s laws.

2. Exercise 11 on page 18 in the text

3. Exercises 2 and 3 on page 22 in the text

4. A coin is tossed as many times as necessary to turn up one head. Thus, the elements of the sample space \(C\) corresponding to this experiment are
   \[H, TH, TTH, TTTTH, \ldots\]
   Let \(\Pr\) be a functions that assigns to these elements the values \(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots\) respectively.

   (a) Show that \(\Pr(C) = 1.\)
   (b) Let \(E_1\) denote the event \(E_1 = \{H, TH, TTH, TTTTH\text{ or } TTTTTH\}\), and compute \(\Pr(E_1).\)
   (c) Let \(E_2 = \{TTTTTH, TTTTTH\}\), and compute \(\Pr(E_2), \Pr(E_1 \cap E_2)\) and \(\Pr(E_2 \setminus E_1)\)

5. Let \(C = \{x \in \mathbb{R} \mid x > 0\}\) and define \(\Pr\) on open intervals \((a, b)\) with \(0 < a < b\) by
   \[\Pr((a, b)) = \int_{a}^{b} e^{-x} \, dx.\]

   (a) Show that \(\Pr(C) = 1.\)
   (b) Let \(E = \{x \in C \mid 4 < x < \infty\}\), and compute \(\Pr(E), \Pr(E^c)\) and \(\Pr(E \cup E^c).\)