Assignment #6

Due on Wednesday February 13, 2008

Read Section 2.1 on *The Definition of Conditional Probability,* pp. 49–18, in DeGroot and Schervish.
Read Section 2.2 on *Independent Events,* pp. 56–64, in DeGroot and Schervish.
Read Section 2.3 on *Bayes' Theorem,* pp. 66–77, in DeGroot and Schervish.

Do the following problems

1. Exercise 2 on page 77 in the text

2. Let $E_1, E_2, \ldots E_k$ be independent events with probabilities $p_1, p_2, \ldots, p_k$, respectively. Show that the probability of at least one of $E_1, E_2, \ldots E_k$ occurring is

$$1 - (1 - p_1)(1 - p_2)\cdots(1 - p_k).$$

3. Suppose a fair die is rolled six independent times. A match occurs if side i is observed in the ith trial, $i = 1, 2, \ldots, 6$.

   (a) What is the probability of at least one match in the six rolls?

   (b) Extend the result of part (a) to a fair n–sided die with n independent rolls. What is the limit of the probability as $n \to \infty$?

4. A die is cast independently until a 6 appears. If the casting stops on an odd number of times, Jane wins; otherwise, Bob wins.

   (a) Assume the die is fair. What is the probability that Jane wins?

   (b) Let p denote the probability of a 6. Show that the game favors Jane for all values of p, $0 < p < 1$.

5. A person answers each of two multiple choice questions at random. If there are four possible choices on each question, what is the conditional probability that both answers are correct given that at least one is correct?