Assignment #8

Due on Wednesday February 20, 2008

Read Section 3.2 on *Continuous Distributions,* pp. 103–108, in DeGroot and Schervish.

Do the following problems

1. A point is selected at random form the sample space $\mathcal{C} = \{x \in \mathbb{R} \mid 0 < x < 10\}$. For any Borel subset $E \subseteq \mathcal{C}$ the probability of E is defined to be
$$\Pr(E) = \int_E \frac{1}{10}\, \mathrm{d}x.$$
Define $X \colon \mathcal{C} \to \mathbb{R}$ to be
$$X(x) = x^2 \quad \text{for all} \quad x \in \mathcal{C}.$$
Find the cumulative distribution function and the probability density function of X.

2. Let $\mathcal{C} = \{x \in \mathbb{R} \mid 0 < x < \infty\}$ and $\mathcal{B}$ denote the Borel sets in $\mathcal{C}$. Let the pdf of a random variable, X, defined on $\mathcal{C}$ be given by
$$f_X(x) = e^{-x} \quad \text{for all} \quad x > 0.$$
Let $E_k = \{x \in \mathcal{C} \mid 2 - 1/k < x \leqslant 3\}$ for $k = 1, 2, 3, \ldots$
Compute $\Pr(E_n)$ for all n, and $\lim_{n\to\infty} \Pr(E_n)$.

3. Exercise 2 on page 109 in the text

4. Exercise 4 on page 109 in the text

5. A *median* of the distribution of a random variable X is a value m for x such that
$$\Pr(X < m) \leqslant \frac{1}{2} \quad \text{and} \quad \Pr(X \leqslant m) \geqslant \frac{1}{2}.$$
If there is only one such value m, it is called *the median of the distribution*.
Suppose the pdf of a random variable X is given by the function
$$f(x) = \begin{cases} \frac{1}{8}x & \text{for} \quad 0 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$
Compute a median for the distribution of X. Is it <u>the</u> median of the distribution?