## Assignment #9

**Due on Wednesday, April 22, 2009**

**Read** Chapter 4 on *Differential Forms,* pp. 77–110, in Bressoud.

**Read** Section 5.4 on *Multiple Integrals,* pp. 120–134, in Bressoud.

**Do** the following problems

1. Exercise 2 on page 86 in the text.

2. Exercises 3 and 4 on pages 86 and 87 in the text.

3. Exercises 1(b) and 1(d) on page 96 in the text.

4. Show that the directed line segment $[P_1, P_2]$ is the smallest convex set that contains the points $P_1$ and $P_2$ in $\mathbb{R}^2$; that is, if $A$ is any convex set in $\mathbb{R}^2$ which contains the points $P_1$ and $P_2$, then
$$[P_1, P_2] \subseteq A.$$

5. Let $P_1$, $P_2$ and $P_3$ be three non–collinear points in $\mathbb{R}^2$. Show that the oriented triangle $T = [P_1, P_2, P_3]$ is the set
$$T = \{\alpha\overrightarrow{OP_1} + \beta\overrightarrow{OP_2} + \gamma\overrightarrow{OP_3} \mid \alpha \geqslant 0, \beta \geqslant 0, \gamma \geqslant 0, \text{ and } \alpha + \beta + \gamma = 1\},$$
where $O$ denotes the origin in $\mathbb{R}^2$. The expression
$$\alpha\overrightarrow{OP_1} + \beta\overrightarrow{OP_2} + \gamma\overrightarrow{OP_3},$$
where $\alpha$, $\beta$ and $\gamma$ are positive real numbers which add up to 1 is called a *convex combination* of the vectors $\overrightarrow{OP_1}$, $\overrightarrow{OP_2}$ and $\overrightarrow{OP_3}$.

6. Let $P$ and $Q$ denote $C^1$ scalar fields defined in some open region, $D$, or $\mathbb{R}^2$, and define the 1–form
$$\omega = P \, \mathrm{d}y - Q \, \mathrm{d}x.$$
   (a) Compute the differential, $\mathrm{d}\omega$, of $\omega$.

(b) Recall that the integral $\int_C \omega$, where $C$ is a simple closed curve in $D$, gives the flux of the field

$$F = P\,\widehat{i} + Q\,\widehat{j}$$

across the curve $C$.

What does the Fundamental Theorem of Calculus,

$$\int_T \mathrm{d}\omega = \int_{\partial T} \omega,$$

where $T$ is a positively oriented triangle in $D$, say about the divergence of $F$ and its flux across the boundary of $T$?

7. Consider the iterated integral

$$\int_0^1 \int_y^1 e^{-x^2}\ \mathrm{d}x\ \mathrm{d}y.$$

(a) Identify the region of integration, $R$, for this integral and sketch it.

(b) Change the order of integration in the iterated integral and evaluate the double integral

$$\int_R e^{-x^2}\ \mathrm{d}x\ \mathrm{d}y.$$

8. Exercise 2 on page 135 in the text.

9. Exercise 3 on page 135 in the text.

10. Exercise 4 on page 135 in the text.