Assignment #10

Due on Friday, February 27, 2009

Read Section 3.5 on Dimension in Messer (pp. 114–121).

Background and Definitions

(Definition of dimension of a subspace of $\mathbb{R}^n$). Let $W$ be a subspace of $\mathbb{R}^n$. The dimension of $W$, denoted by $\dim(W)$, is the number of vectors in any basis for $W$.

Do the following problems

1. Let

$$W_1 = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid x + y - z = 0 \right\} \quad \text{and} \quad W_2 = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid x + 2y + z = 0 \right\}.$$

Find a bases for $W_1$ and $W_2$ and compute $\dim(W_1)$ and $\dim(W_2)$.

2. Let $W_1$ and $W_2$ be as defined in Problem 1. Find a basis for $W_1 \cap W_2$ and compute $\dim(W_1 \cap W_2)$.

3. Let $W_1$ and $W_2$ be as defined in Problem 1. Find a basis for $W_1 + W_2$ and compute $\dim(W_1 + W_2)$.

Use the results of Problems 1 and 2 to verify that

$$\dim(W_1 + W_2) = \dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2).$$

4. Let $A = \begin{pmatrix} 1 & -2 & -3 & 0 \\ -1 & 0 & 2 & 1 \\ 1 & 4 & 0 & -3 \end{pmatrix}$.

(a) Find a basis for the column space, $C_A$, of the matrix $A$ and compute $\dim(C_A)$.

(b) Find a basis for the null space, $N_A$, of the matrix $A$ and compute $\dim(N_A)$.

(c) Compute $\dim(N_A) + \dim(C_A)$. What do you observe?

5. Let $A$ denote the matrix defined in the previous problem. Consider the rows of $A$ as row vectors in $\mathbb{R}^4$, and let $R_A$ denote the span of the rows of the matrix $A$. Find a basis for $R_A$, and compute $\dim(R_A)$. What do you find interesting about $\dim(R_A)$ and $\dim(C_A)$, which was computed in the previous problem.