Assignment #22

Due on Friday, April 24, 2009

Read Section 6.1 on Linear Functions in Messer (pp. 212–216).
Read Section 6.3 on Matrix of a Linear Function in Messer (pp. 226–231).
Read Section 6.2 on Compositions and Inverses in Messer (pp. 218–223).

Do the following problems

1. Let $T: \mathbb{R}^n \to \mathbb{R}^n$ denote a linear transformation and $I$ denote the identity transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$. For scalars $a$ and $b$, prove the following:
   (a) $T$ and $T - aI$ commute; that is,
   $$T \circ (T - aI) = (T - aI) \circ T;$$
   (b) $T - aI$ and $T - bI$ commute.

2. Let $R_\theta: \mathbb{R}^2 \to \mathbb{R}^2$ denote rotation around the origin in $\mathbb{R}^2$ in the counterclockwise sense trough and angle of $\theta$. Show that $R_\theta$ is invertible and compute its inverse.

3. Let $R_\theta: \mathbb{R}^2 \to \mathbb{R}^2$ denote rotation around the origin in $\mathbb{R}^2$ in the counterclockwise sense through an angle of $\theta$, and $R_\phi$ denote a similar rotation through an angle of $\phi$.
   (a) Show that the composition $R_\theta \circ R_\phi$ is also a rotation in $\mathbb{R}^2$. What is the angle of rotation in for the composite rotation?
   (b) Show that $R_\theta$ and $R_\phi$ commute.

4. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ denote reflection across the line $y = x$. Express $T$ as a composition of rotations and a reflection across the $x$–axis.

5. Let $T_1: \mathbb{R}^2 \to \mathbb{R}^2$ denote reflection across the line $y = x$ and $T_2: \mathbb{R}^2 \to \mathbb{R}^2$ denote reflection across the $y$–axis.
   (a) Show that $T_2 \circ T_1$ is a rotation in $\mathbb{R}^2$. What is the angle of rotation?
   (b) What do you get if you compose $T_1 \circ T_2$?