Assignment #22

Due on Friday, April 24, 2009

Read Section 6.1 on *Linear Functions* in Messer (pp. 212–216).

Read Section 6.3 on *Matrix of a Linear Function* in Messer (pp. 226–231).

Read Section 6.2 on *Compositions and Inverses* in Messer (pp. 218–223).

Do the following problems

1. Let $T\colon \mathbb{R}^n \to \mathbb{R}^n$ denote a linear transformation and I denote the identity transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$. For scalars a and b, prove the following:

   (a) T and $T - aI$ commute; that is,
   $$T \circ (T - aI) = (T - aI) \circ T;$$

   (b) $T - aI$ and $T - bI$ commute.

2. Let $R_\theta\colon \mathbb{R}^2 \to \mathbb{R}^2$ denote rotation around the origin in $\mathbb{R}^2$ in the counterclockwise sense trough and angle of θ. Show that R_θ is invertible and compute its inverse.

3. Let $R_\theta\colon \mathbb{R}^2 \to \mathbb{R}^2$ denote rotation around the origin in $\mathbb{R}^2$ in the counterclockwise sense through an angle of θ, and R_φ denote a similar rotation through an angle of φ.

   (a) Show that the composition $R_\theta \circ R_\varphi$ is also a rotation in $\mathbb{R}^2$. What is the angle of rotation in for the composite rotation?

   (b) Show that R_θ and R_φ commute.

4. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ denote reflection across the line $y = x$. Express T as a composition of rotations and a reflection across the x–axis.

5. Let $T_1\colon \mathbb{R}^2 \to \mathbb{R}^2$ denote reflection across the line $y = x$ and $T_2\colon \mathbb{R}^2 \to \mathbb{R}^2$ denote reflection across the y–axis.

   (a) Show that $T_2 \circ T_1$ is a rotation in $\mathbb{R}^2$. What is the angle of rotation?

   (b) What do you get if you compose $T_1 \circ T_2$?