Assignment #3

Due on Wednesday, February 4, 2009

Read Section 3.3 on Linear Independence in Messer (pp. 103–109).

Do the following problems

1. Consider the vectors \( v_1, v_2 \) and \( v_3 \) in \( \mathbb{R}^3 \) given by

\[
\begin{align*}
    v_1 &= \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, &
    v_2 &= \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}, &
    v_3 &= \begin{pmatrix} 0 \\ -4 \\ 3 \end{pmatrix}.
\end{align*}
\]

(a) If possible, write the vector \( v_3 \) as a linear combination of \( v_1 \) and \( v_2 \).
(b) Determine whether the set \( \{v_1, v_2, v_3\} \) spans \( \mathbb{R}^3 \).

2. Let \( v_1, v_2 \) and \( v_3 \) be as given in the previous problem. Find a linearly independent subset of \( \{v_1, v_2, v_3\} \) which spans \( \text{span}\{v_1, v_2, v_3\} \).

3. Show that the set \( \left\{ \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \right\} \) is a linearly independent subset of \( \mathbb{R}^3 \).

4. Determine whether the set \( \left\{ \begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ -1 \\ 0 \end{pmatrix} \right\} \) is a linearly independent subset of \( \mathbb{R}^4 \).

5. Show that \( \left\{ \begin{pmatrix} 2 \\ 2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 3 \\ -2 \end{pmatrix} \right\} \) is a linearly dependent subset of \( \mathbb{R}^4 \). Write one of the vectors in the set as a linear combination of the other three. Show that the remaining three vectors form a linearly independent subset of \( \mathbb{R}^4 \).