Exam 1

March 6, 2009

Name: __________________________________________

This is a closed book exam. Show all significant work and justify all your answers. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 4 problems. Relax.

1. Answer the following questions as thoroughly as possible.
   (a) State precisely what it means for the set of vectors \( \{v_1, v_2, \ldots, v_k\} \) in \( \mathbb{R}^n \) to be linearly independent.
   (b) Define the span of the set of vectors, \( S \), in \( \mathbb{R}^n \).
   (c) Let \( W \) denote a subspace of \( \mathbb{R}^n \). Define the coordinates of a vector \( v \in W \) relative to a basis \( B \) for \( W \).

2. Determine whether the following statements are true or false. If false, give examples to justify your conclusion. If true, provide an argument to justify your answer.
   (a) The set, \( \{v_1, v_2, v_3\} \), of vectors in \( \mathbb{R}^2 \) is linearly dependent.
   (b) The set of vectors in \( \mathbb{R}^3 \), \( \{0, v_1, v_2\} \) is linearly independent.
   (c) If \( S_1 \) and \( S_2 \) are linearly independent, then \( S_1 \cup S_2 \) is also linearly independent.

3. Let \( \langle v, w \rangle \) denote the Euclidean inner product in \( \mathbb{R}^n \). For a fixed vector \( u \) in \( \mathbb{R}^n \), define the set
   \[ W = \{w \in \mathbb{R}^n \mid \langle u, w \rangle = 0\}. \]
   Prove that \( W \) is a subspace of \( \mathbb{R}^n \).

4. Find a basis for the solution space, \( W \), of the homogenous system
   \[
   \begin{align*}
   3x_1 - x_2 + 2x_3 + x_4 &= 0 \\
   2x_1 - x_2 + x_3 &= 0 \\
   x_1 + x_3 + x_4 &= 0,
   \end{align*}
   \]
   and compute \( \dim(W) \).