Assignment #13

Due on Monday, April 12, 2010

Do the following problems.

1. Let \((x_n)\) denote a sequence of real numbers and \((x_{n_k})\) denote a subsequence of \((x_n)\).

   (a) Prove that if \((x_n)\) converges then \((x_{n_k})\) converges.
   (b) Show that the converse of the statement proved in part (a) is not true.

In Problems 2-5, you will prove the Binomial Theorem: for any \(a, b \in \mathbb{R}\),

\[
(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k},
\]

where

\[
\binom{n}{k} = \frac{n!}{k!(n-k)!}, \text{ for } k = 0, 1, 2, \ldots, n,
\]

are called the binomial coefficients.

2. Use the formula \((k + 1)! = (k + 1) \cdot k!\) to establish that \(0! = 1\), and compute \(\binom{m}{0}\) and \(\binom{m}{m}\) for all \(m \in \mathbb{N}\).

3. Prove that \(\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}\) for all \(n \in \mathbb{N}\) and all \(k = 1, \ldots, n\).

4. Use induction to prove that, for any real number, \(x\),

\[
(1 + x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k.
\]

5. Use the expansion in (2) to deduce the expansion in (1) for any real numbers \(a\) and \(b\).