Assignment #15

Due on Wednesday, April 21, 2010

Do the following problems.

1. Let $m$ denote a natural number and define $x_n = \frac{1}{n^m}$ for all $n \in \mathbb{N}$. Prove that $(x_n)$ converges to 0 as $n \to \infty$.

2. Let $q$ denote a positive rational number and define $x_n = \frac{1}{n^q}$ for all $n \in \mathbb{N}$. Prove that $(x_n)$ converges to 0 as $n \to \infty$.

3. Let $(x_n)$ denote a sequence of nonnegative real numbers. Suppose that $(x_n)$ converges to $a$ as $n \to \infty$. Prove that $a \geq 0$ and that $(\sqrt{x_n})$ converges to $\sqrt{a}$.

4. Let $x_n = \sqrt{\frac{n+1}{n}}$ for all $n \in \mathbb{N}$. Prove that the sequence $(x_n)$ converges and compute its limit.

5. Let $x_n = \sqrt{n^2 + n} - n$ for all $n \in \mathbb{N}$. Determine if the sequence $(x_n)$ converges or not. If it converges, compute its limit.

*Suggestion:* Consider the product $(\sqrt{n^2 + n} + n)x_n$ for each $n \in \mathbb{N}$. 