Assignment #16

Due on Monday, April 26, 2010

Do the following problems.

1. Let $a > 0$.

   (a) Use the Principle of Mathematical Induction to show how to define a^n for all $n \in \mathbb{N}$.

   (b) Prove that $a^{m+n} = a^m a^n$ for all $m, n \in \mathbb{N}$.

2. Let $a > 0$.

   (a) Explain how to define a^0 and a^{-1}.

   (b) For each $n \in \mathbb{Z}$, explain how to define a^n.

   (c) Prove that $a^{m+n} = a^m a^n$ for all $m, n \in \mathbb{Z}$.

   (d) Prove that $a^{m-n} = \dfrac{a^m}{a^n}$ for all $m, n \in \mathbb{Z}$.

3. Let $a > 0$ and $m \in \mathbb{N}$. Prove that the equation $x^m = a$ has a unique positive solution.

4. Let $a > 0$.

   (a) Use the result of Problem 3 to explain how to define $a^{1/m}$ for $m \in \mathbb{N}$.

   (b) Explain how to define $a^{n/m}$ for $m \in \mathbb{N}$ and $n \in \mathbb{Z}$.

   (c) For rational numbers, q and r, prove that $a^{q+r} = a^q a^r$.

   (d) For rational numbers, q and r, prove that $a^{q-r} = \dfrac{a^q}{a^r}$.

5. Suppose that $a > 1$.

   (a) Let $n, m \in \mathbb{N}$. Prove that $m < n \Rightarrow a^m < a^n$.

   (b) Let $n, m \in \mathbb{N}$. Prove that $m < n \Rightarrow a^{1/n} < a^{1/m}$.

   (c) Let $q, r \in \mathbb{Q}$. Prove that $0 < q < r \Rightarrow a^q < a^r$.