

Assignment #3

Due on Wednesday, February 3, 2010

Read Handout #2 on *The Real Numbers System Axioms*.

Read Section 3.1 on *The Rules of Arithmetic* on pp. 43–44 in Schramm's text.

Read Section 3.2 on *Fields* on pp. 44–46 in Schramm's text.

Do the following problems

Use the Field Axioms in Handout #2 to prove the following:

1. Let x denote a real number satisfying $x^2 = x$. Prove that either $x = 0$ or $x = 1$.
(Note that $x^2 = xx$.)

2. Let $a \in \mathbb{R}$. Prove that if $a \neq 0$, then the equation

$$ax = b$$

has a unique solution for every $b \in \mathbb{R}$.

3. Let $x \in \mathbb{R}$. Prove that $(-1)x$ is the additive inverse of x ; that is $x + (-1)x = 0$.
4. Prove that, for any real number, x ,

$$(-x)^2 = x^2.$$

5. Let $a, b \in \mathbb{Q}$, where $a^2 + b^2 \neq 0$.

(a) Explain by $a^2 - 2b^2 \neq 0$.

(b) Show that the multiplicative inverse of $a + b\sqrt{2}$, namely $(a + b\sqrt{2})^{-1}$, is of the form $c + d\sqrt{2}$, where $c, d \in \mathbb{Q}$.