Assignment #6

Due on Monday, February 15, 2010

Read Handout #2 on The Real Numbers System Axioms.
Read Section 4.6 on Ordered Fields on pp. 63–66 in Schramm’s text.
Read Section 4.7 on Absolute Value and Distance on pp. 68–68 in Schramm’s text.

Do the following problems

1. Let $x \in \mathbb{R}$. Prove that $0 < x \leq 1$ implies that $x^2 \leq x$.

2. Let $a$ and $b$ denote real numbers. Use the triangle inequality to prove that

$$||a| - |b|| \leq |a - b|.$$ 

3. Let $a$ and $b$ denote positive real numbers. Start with the true statement

$$(a - b)^2 \geq 0$$

to prove the inequality

$$ab \leq \frac{a^2 + b^2}{2}.$$ 

Prove that equality holds if and only if $a = b$.

4. Given a real number $x$, denote by $\max\{x, 0\}$ the larger of $x$ and 0. Prove that

$$\max\{x, 0\} = \frac{x + |x|}{2}.$$ 

5. Let $x$ and $\max\{x, 0\}$ be as in the previous problem. Denote by $\min\{x, 0\}$ the smaller of $x$ and 0. Prove that

$$\min\{x, 0\} = -\max\{-x, 0\},$$

and use this result to derive a formula for $\min\{x, 0\}$ analogous to that for $\max\{x, 0\}$ proved in the previous problem.