Assignment #7

Due on Monday, March 1, 2010

Read Handout #2 on *The Real Numbers System Axioms.*

Read Section 4.6 on *Ordered Fields* on pp. 63–66 in Schramm's text.

Read Chapter 5 on *Upper Bounds and Suprema,* pp. 80–85, in Schramm's text.

Do the following problems

1. Let $a, b \in \mathbb{R}$. Prove that
$$a < b \text{ if and only if } a < \frac{a+b}{2} < b.$$

2. Prove that between any two rational numbers there is at least one rational number.

3. Prove that between any two rational numbers there are infinitely many rational numbers.

4. Given two subsets, A and B, of real numbers, the union of A and B is the set $A \cup B$ defined by
$$A \cup B = \{x \in \mathbb{R} \mid x \in A \text{ or } x \in B\}$$
Assume that A and B are non–empty and bounded above. Prove that $\sup(A \cup B)$ exists and
$$\sup(A \cup B) = \max\{\sup(A), \sup(B)\},$$
where $\max\{\sup(A), \sup(B)\}$ denotes the largest of $\sup(A)$ and $\sup(B)$.

5. Given two subsets, A and B, of real numbers, the intersection of A and B is the set $A \cap B$ defined by
$$A \cap B = \{x \in \mathbb{R} \mid x \in A \text{ and } x \in B\}$$
Is it true that $\sup(A \cap B) = \min\{\sup(A), \sup(B)\}$?
Here, $\min\{\sup(A), \sup(B)\}$ denotes the smallest of $\sup(A)$ and $\sup(B)$.