Assignment #9

Due on Monday, March 8, 2010

Read Handout #2 on *The Real Numbers System Axioms.*

Read Section 4.6 on *Ordered Fields* on pp. 63–66 in Schramm's text.

Read Chapter 5 on *Upper Bounds and Suprema,* pp. 80–85, in Schramm's text.

Do the following problems

1. Let $a, b \in \mathbb{R}$. Show that if $a \leqslant b + \varepsilon > 0$ for all $\varepsilon > 0$, then $a \leqslant b$.

   (Note that this problem is very similar to Problem 6 in Problem Set #2, but it is not the same problem).

2. Let x denote a positive real number. Prove that $0 < z < 1$ implies that $zx < x$.

3. Let A be a non–empty and bounded subset of $\mathbb{R}$. Prove that

   $$\inf(A) \leqslant \sup(A).$$

4. Let A and B be a non–empty subsets of $\mathbb{R}$ which are bounded from above. Prove that if $\sup A < \sup B$, then there exists $b \in B$ such that b is an upper bound for A.

5. Use the fact that between any two distinct real numbers there is a rational number to prove the statement:

   Between any two distinct real numbers there is at least an irrational number.