Exam 2 (Part I)

Friday, April 2, 2010

Name: ________________________________

Provide complete arguments when asked to prove a statement in a question. You will be graded on how well you organize your proofs as well as the logical flow or your deductions. This is a closed–book, closed–notes exam. Use your own paper and/or the paper provided for you. Write your name on this page and staple it to your solutions. You have 50 minutes to work on the following 3 problems. Relax.

1. Let \((x_n)\) denote a sequence of real numbers.
   (a) State precisely what the statement “\((x_n)\) converges” means.
   (b) Let \(x_n = \frac{1}{\sqrt{n}}\) for all \(n \in \mathbb{N}\). Use the definition that you stated in the previous part to prove that \((x_n)\) converges.

2. Let \((x_n)\) denote a sequence of real numbers.
   (a) State precisely what it means for \((x_n)\) to be a Cauchy sequence.
   (b) Prove that if \((x_n)\) converges, the it is a Cauchy sequence.

3. Let \(B \subseteq \mathbb{R}\) be a non–empty subset which is bounded below and put \(\ell = \inf B\).
   (a) Prove that there exists a sequence of numbers in \(B\) which converges to \(\ell\).
   (b) Apply the result of the previous part to the set
   \[
   B = \{q \in \mathbb{Q} \mid q > 0 \text{ and } q^2 > 2\}
   \]
   to deduce that there exists a sequence of rational numbers \(\{q_n\}\) which converges to \(\sqrt{2}\).

Note: You will need to prove that \(\inf B = \sqrt{2}\).