Exam 2 (Part II)

Friday, April 2, 2010

Name: ______________________________

This is the out–of–class portion of Exam 2. There is no time limit for working on the following two problems. This is a closed–book, closed–notes exam.
Provide complete arguments when asked to prove a statement in a question. You will be graded on how well you organize your proofs as well as the logical flow or your deductions.
Write your name on this page and staple it to your solutions.

Due on Monday, April 5, 2010

1. Let (x_n) denote a sequence of real numbers. For a fixed $N_o \in \mathbb{N}$, define

$$y_n = x_{_{N_o+n}} \quad \text{for all } n \in \mathbb{N};$$

that is; $y_1 = x_{_{N_o+1}}$, the $(N_o+1)^{\text{th}}$ term in the sequence (x_n), y_2 is the $(N_o+2)^{\text{th}}$ term, and so on.

   (a) Prove that (x_n) converges if and only if (y_n) converges.

   (b) Prove that if (x_n) is bounded and (y_n) is monotone, the both (x_n) and (y_n) converge.

   (c) Give an interpretation of the results in this problem.

2. Define a sequence, (x_n), of real numbers as follows:

$$\begin{aligned} x_1 &= 1; \\ x_{n+1} &= \sqrt{1+x_n} \quad \text{for all } n \in \mathbb{N}. \end{aligned}$$

   (a) Prove that (x_n) is monotone.
   Suggestion: Consider $x_{n+2}^2 - x_{n+1}^2$

   (b) Show that $x_n < 2$ for all $n \in \mathbb{N}$.

   (c) Deduce that (x_n) converges.

   (d) Compute the limit of (x_n).